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Existence and uniqueness of dynamic evolutions for a peeling test in dimension one

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Abstract

In this paper we present a one-dimensional model of a dynamic peeling test for a thin film, where the wave equation is coupled with a Griffith criterion for the propagation of the debonding front. Our main results provide existence and uniqueness for the solution to this coupled problem under different assumptions on the data.

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Keywords: Dynamic debonding; Wave equation in time-dependent domains; Dynamic energy release rate; Maximum dissipation principle; Griffith's criterion; Dynamic fracture

0. Introduction

The study of crack growth based on Griffith's criterion has become of great interest in the mathematical community. The starting point was the seminal paper [15], where a precise variational scheme for the quasistatic evolution has been proposed. This strategy has been exploited under different hypotheses in [12,6,14,7,10,23,25]. The approximation of brittle crack growth by means of phase-field models in the quasistatic regime has been studied in [18]. A comprehensive

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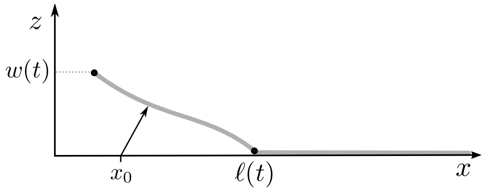

Fig. 1. The curve $x \mapsto (x + h(t, x), u(t, x))$ describing the deformation of the film in the peeling test. The vector applied to the point x_0 is $(h(t, x_0), u(t, x_0))$.

presentation of the variational approach to quasistatic fracture mechanics can be found in [5]. For the relationships between this approach and the general theory of rate-independent systems we refer to the recent book [29].

In the dynamic case no general formulation has been yet proposed and only preliminary results are available (see [30,8,11,9]). A reasonable model for dynamic fracture should combine the equations of elasto-dynamics for the displacement u out of the crack with an evolution law which connects the crack growth with u. The only result in this direction, without strong geometrical assumptions on the cracks, has been obtained for a phase-field model [24], but the convergence of these solutions to a brittle crack evolution has not been proved in the dynamic case. In the latter model the equation of elasto-dynamics for u is coupled with a suitable minimality condition for the phase-field ζ at each time. Other models in materials science, dealing with damage or delamination, couple a second order hyperbolic equation for a function u with a first order flow rule for an internal variable ζ (see, e.g., [16,4,3,31,32,21,20] for viscous flow rules on ζ and [34,36,35,33,37,2,1,38,27,28] for rate-independent evolutions of ζ).

In this work we contribute to the study of dynamic fracture by analysing a simpler one-dimensional model already considered in [17, Section 7.4]. This model exhibits some of the relevant mathematical difficulties due to the time dependence of the domain of the wave equation. More precisely, following [13,26] we study a model of a dynamic peeling test for a thin film, initially attached to a planar rigid substrate; the process is assumed to depend only on one variable. This hypothesis is crucial for our analysis, since we frequently use d'Alembert's formula for the wave equation.

To describe the geometry of our problem, we fix an orthogonal coordinate system (x, y, z). We assume that the z-axis is vertical, the plane (x, y) coincides with the rigid substrate, and that the reference configuration of the film is the half plane $\{(x, y) : x \geq 0\}$. We also assume that the deformation of the film at time $t \geq 0$ is described by two functions h and u according to the formula $(x, y) \mapsto (x + h(t, x), y, u(t, x))$, i.e., the displacement is given by $(h(t, x), 0, u(t, x))$. Therefore the thin film at time t is uniquely determined by the parametric curve $x \mapsto (x + h(t, x), u(t, x))$ with $x \geq 0$, which represents its intersection with the vertical plane (x, z) (see Fig. 1). To simplify our analysis we shall not consider the unilateral contact constraint $u(t, x) \geq 0$, thus neglecting the non-interpenetration of matter.

The film is assumed to be perfectly flexible, inextensible, and glued to the rigid substrate on the half line $\{(x, y, z) : x \geq \ell(t), z = 0\}$, where $\ell(t)$ is a nondecreasing function which represents the debonding front, with $\ell_0 := \ell(0) > 0$. This implies $h(t, x) = u(t, x) = 0$ for $x \geq \ell(t)$. At $x = 0$ we prescribe a vertical displacement $u(t, 0) = w(t)$ depending on time $t \geq 0$, and a fixed tension so that the speed of sound in the film is constant. Using the linear approximation and the inextensibility it turns out that h can be expressed in terms of u as