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Boundary layer problem on a hyperbolic system arising from chemotaxis

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Abstract

This paper is concerned with the boundary layer problem for a hyperbolic system transformed via a Cole–Hopf type transformation from a repulsive chemotaxis model with logarithmic sensitivity proposed in [23,34] modeling the biological movement of reinforced random walkers which deposit a non-diffusible (or slowly moving) signal that modifies the local environment for succeeding passages. By prescribing the Dirichlet boundary conditions to the transformed hyperbolic system in an interval $(0, 1)$, we show that the system has the boundary layer solutions as the chemical diffusion coefficient $\varepsilon \to 0$, and further use the formal asymptotic analysis to show that the boundary layer thickness is $\varepsilon^{1/2}$. Our work justifies the boundary layer phenomenon that was numerically found in the recent work [25]. However we find that the original chemotaxis system does not possess boundary layer solutions when the results are reverted to the pre-transformed system.
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1. Introduction

Chemotaxis is a common phenomenon in biology describing the change of motion of species in response to a chemical stimulus spread in the environment. The consequence of chemotaxis is that the species changes its movement toward (or away from) a higher concentration of the chemical stimulus. The first chemotaxis model was derived by Keller and Segel in a series of works [20–22] to describe abundant biological processes including the aggregation phase of cellular slime mold and traveling waves formed by bacterial chemotaxis. Since then, numerous variants of the Keller–Segel model have been developed to interpret the various biological phenomena/processes, such as aggregative patterns of bacteria [33,44], slime mould formation [14], fish pigmentation patterning [35], angiogenesis in tumor progression [4], primitive streak formation [36], blood vessel formation [9], wound healing [38], and so on. The Keller–Segel model, in its general form, reads

$$\begin{cases} n_t = [Dn_x - \chi n\phi(c)_x]_x, \\ c_t = \varepsilon c_{xx} + g(n, c), \end{cases} \tag{1.1}$$

where $n(x,t)$ and $c(x,t)$ denote cell density and chemical concentration, respectively. The parameter $D > 0$ is the diffusivity of endothelial cells, χ is the chemotactic coefficient and $\varepsilon \geq 0$ denotes the chemical diffusion rate. The chemotaxis is said to be attractive if $\chi > 0$ and repulsive if $\chi < 0$ with $|\chi|$ measuring the intensity of chemotaxis. The function $\phi(c)$ is commonly called the chemotactic sensitivity function accounting for the chemical signal detection mechanism and $g(n,c)$ denotes the chemical kinetics.

With $\chi > 0$, $\phi(c) = \ln c$ and $g(n,c) = -knc^m$ $(k > 0, m \geq 0)$, the model (1.1) was well-known as the Keller–Segel model proposed in [20] to describe the traveling band propagation of bacterial chemotaxis observed in the famous experiment of Adler [1,2]. The analytical studies of this model have been continuously undertaken in a series of works (see survey papers [17,46] and references therein). When $\chi > 0$, $\phi(c) = c$ and $g(n,c) = n - c$, the model (1.1) was well-known as classical Keller–Segel model first proposed in [21] to describe the aggregation phase of slime mold amoebae *Dictyostelium discoideum*, which has attracted extensive attentions in the past few decades (see survey articles [3,13,16]). In contrast to the attractive chemotaxis models, the studies of repulsive chemotaxis (i.e. $\chi < 0$) are much less and not many results have been developed. It is generally believed that repulsive chemotaxis is a stabilizing factor for the dynamics, but its mathematical mechanism has not been completely understood (see [52]). In this paper, we shall consider the following Keller–Segel type repulsive chemotaxis model

$$\begin{cases} n_t = [Dn_x - \chi n(\ln c)_x]_x, \\ c_t = \varepsilon c_{xx} + nc - \mu c, \end{cases} \tag{1.2}$$

where $\chi < 0$. This model was developed in [23,34] to model the biological movement of reinforced random walkers that deposit a non-diffusible (or slowly moving) substance that modifies the local environment for succeeding passages with little or no transport of the modifying substance.

The characteristic of model (1.2) lies in the logarithmic sensitivity function $\ln c$ which is singular at $c = 0$. This singularity brings great difficulties in analytical studies such as the stability of traveling waves and well-posedness problem. Among other things, the foremost mathematical

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