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Analysis of time-domain scattering by periodic structures ☆

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Abstract

This paper is devoted to the mathematical analysis of a time-domain electromagnetic scattering by periodic structures which are known as diffraction gratings. The scattering problem is reduced equivalently into an initial-boundary value problem in a bounded domain by using an exact transparent boundary condition. The well-posedness and stability of the solution are established for the reduced problem. Moreover, a priori energy estimates are obtained with minimum regularity requirement for the data and explicit dependence on the time.

Keywords: Time-domain Maxwell's equations; Diffraction gratings; Well-posedness and stability; A priori estimates

1. Introduction

This paper is concerned with the mathematical analysis of an electromagnetic scattering problem in periodic structures, where the wave propagation is governed by the time-domain Maxwell equations. The scattering theory in periodic diffractive structures, also known as diffraction grat-

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ings, has applications in many cutting-edge scientific areas including ultra-fast and high-energy lasers, space flight instruments, astronomy, and synchrotron spectrometers. A good introduction can be found in [10] to diffraction grating problems and various numerical approaches. The book [19] contains descriptions of several mathematical problems that arise in diffractive optics modeling in industry. Some more recent developments are addressed in [6] on theory, analysis, and computational techniques of diffractive optics.

The time-harmonic grating problems have been extensively studied by many researchers via either the integral equation methods or the variational methods [2–4,12,17]. A survey may be found in [7] for mathematical studies in rigorous grating theory. The general result may be stated as follows: The diffraction problem has a unique solution for all but a countable sequence of singular frequencies. Unique solvability for all frequencies can be obtained for gratings which have absorbing media or perfectly electrically conducting surfaces with Lipschitz profiles. Numerical methods are developed for both the two-dimensional Helmholtz equation (one-dimensional gratings) and the three-dimensional Maxwell equations (crossed or two-dimensional gratings) [5,8,9,15,30].

The time-domain scattering problems have attracted considerable attention due to their capability of capturing wide-band signals and modeling more general material and nonlinearity [11,22,23,25,29]. Comparing with the time-harmonic problems, the time-domain problems are much less studied due to the additional challenge of the temporal dependence. Rigorous mathematical analysis is very rare. The analysis can be found in [14,28] for the time-domain acoustic and electromagnetic obstacle scattering problems. We refer to [24] for the analysis of the time-dependent electromagnetic scattering from a three-dimensional open cavity. Numerical solutions can be found in [18,27] for the time-dependent wave scattering by periodic structures/surfaces. The theoretical analysis is still lacking for the time-domain scattering by periodic structures.

The goal of this work is to analyze mathematically the time-domain scattering problem which arises from the electromagnetic wave propagation in a periodic structure. Specifically, we consider an electromagnetic plane wave which is incident on a one-dimensional grating in $\mathbb{R}^3$. So the structure is assumed to be invariant in the y-direction and periodic in the x-direction. The three-dimensional Maxwell equations can be decomposed into two fundamental polarizations: transverse electric (TE) polarization and transverse magnetic (TM) polarization, where Maxwell's equations are reduced to the two-dimensional wave equation. We shall study the wave equation in two dimensions for both polarizations. The structure can also be characterized by the medium parameters: the electric permittivity and the magnetic permeability. They are periodic in x and assumed only to be bounded measurable functions. Hence our method works for very general gratings whose surfaces/interfaces are allowed to be Lipschitz profiles or even graphs of some Lipschitz continuous functions.

There are two challenges of the problem: time dependence and unbounded domain. In the frequency domain, various approaches have been developed to truncate unbounded domains into bounded ones, such as absorbing boundary conditions (ABCs), transparent boundary conditions (TBCs), and perfectly matched layer (PML) techniques. These effective boundary conditions are being extended to handle time-domain problems [1,13,20,21]. Utilizing the Laplace transform as a bridge between the time-domain and the frequency domain, we develop an exact time-domain TBC and reduce the problem equivalently into an initial boundary value problem in a bounded domain. Using the energy method with new energy functions, we show the well-posedness and stability of the time-dependent problem. The proofs are based on examining the well-posedness of the time-harmonic Helmholtz equations with complex wavenumbers and applying the abstract inversion theorem of the Laplace transform. Moreover, a priori estimates, featuring an explicit