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Global attractors for a third order in time nonlinear dynamics

Arthur H. Caixeta [a], Irena Lasiecka [b,c], Valéria N.D. Cavalcanti [a,*]

[a] *Department of Mathematics, University of Maringá, Maringá, Brazil*
[b] *Department of Mathematical Sciences, University of Memphis, Memphis, TN 38152, United States*
[c] *IBS, Polish Academy of Sciences, Warsaw, Poland*

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Abstract

Long time behavior of a third order (in time) nonlinear PDE equation is considered. This type of equations arises in the context of nonlinear acoustics [12,20,22,24] where modeling accounts for a finite speed of propagation paradox, the latter results in hyperbolic nature of the dynamics. It will be proved that the underlying PDE generates a well-posed dynamical system which admits a global and finite dimensional attractor. The main difficulty associated with the problem studied is the lack of Lyapunov function along with the lack of compactness of trajectories, which fact prevents applicability of standard tools in the area of dynamical systems.

Contents

* Corresponding author.
E-mail addresses: arthur-caixeta@hotmail.com (A.H. Caixeta), lasiecka@memphis.edu (I. Lasiecka), vndcavalcanti@uem.br (V.N.D. Cavalcanti).

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1. Introduction

This manuscript is devoted to a long time behavior of *third order in time* nonlinear dynamics governed by an abstract differential equation defined on a suitable Hilbert space. This kind of models have recently been considered in the context of modeling waves in nonlinear acoustics [12,24,21] where Fourier's law is replaced by the more realistic Maxwell–Cattaneo's Law, which accounts for a finite speed of propagation of acoustic waves. This leads to three derivatives in time (as opposed to two derivatives in the classical models) [20,26,27]. The presence of three derivatives in time changes completely the character of underlying dynamics – from "parabolic" to "hyperbolic" making it rather rich – with new phenomena taking place and ranging from nonexistence [13,28] of solution to an asymptotic decay of finite energy solutions [16]. In [23], the authors investigate the decay of the energy in the presence of a viscoelastic dissipation.

The model under consideration takes the following abstract form:

$$\tau u_{ttt} + \alpha u_{tt} + c^2 Au + bAu_t = f(u, u_t, u_{tt}) \tag{1}$$

Here A is a self-adjoint, positive operator densely defined on a Hilbert space H and $f(u, u_t, u_{tt})$ is a semilinear term. The coefficient $\tau > 0$ denotes parameter of speed relaxation, it is usually very small as it physically corresponds to relaxation parameter of Maxwell–Cattaneo's law. In various applications, this follows from replacement of a standard Fourier's law for the heat flux by Maxwell–Cattaneo's law. This particular model, in the above generality, includes Moore–Gibson–Thompson (MGT) equation which arises in the context of modeling High Frequency Ultrasound waves. In this case, the variable u denotes acoustic pressure (or also acoustic velocity potential), the operator A denotes $-\Delta$ with either Dirichlet or Neumann boundary conditions, $c > 0$ is the speed of sound, $b = \delta + \tau c^2$ where $\delta > 0$ is the diffusivity of the sound. The constant α represents frictional damping in the model. There are in the literature several relevant models describing propagation on nonlinear acoustic waves (see [18,33]). One of the commonly used is the MGT model where

$$f(u, u_t, u_{tt}) = k\frac{d^2}{dt^2}u^2 \tag{2}$$

with $k = \frac{1+B}{\rho c^2}$, B denoting the parameter of nonlinearity and ρ stands for density. The model under consideration in (1) is third order in time with a small parameter τ, denoting relaxation parameter, which results from accounting in the model for finite speed of propagation of acoustic