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Regularity criterion for the 3D Hall-magneto-hydrodynamics

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Abstract

This paper studies the regularity problem for the 3D incompressible resistive viscous Hall-magneto-hydrodynamic (Hall-MHD) system. The Kolmogorov 41 phenomenological theory of turbulence [14] predicts that there exists a critical wavenumber above which the high frequency part is dominated by the dissipation term in the fluid equation. Inspired by this idea, we apply an approach of splitting the wavenumber combined with an estimate of the energy flux to obtain a new regularity criterion. The regularity condition presented here is weaker than conditions in the existing criteria (Prodi–Serrin type criteria) for the 3D Hall-MHD system.
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1. Introduction

In this paper we consider the three dimensional incompressible resistive viscous Hall-magneto-hydrodynamics (Hall-MHD) system:

$$\begin{aligned} u_t + u\cdot\nabla u - b\cdot\nabla b + \nabla p &= \nu\Delta u,\\ b_t + u\cdot\nabla b - b\cdot\nabla u + \nabla\times((\nabla\times b)\times b) &= \mu\Delta b,\\ \nabla\cdot u &= 0, \end{aligned} \tag{1.1}$$

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with the initial conditions

$$u(x,0)=u_0(x), \qquad b(x,0)=b_0(x), \qquad \nabla\cdot u_0=\nabla\cdot b_0=0, \tag{1.2}$$

where $x\in\mathbb{R}^3$, $t\geq 0$, u is the fluid velocity, p is the fluid pressure and b is the magnetic field. The parameter ν denotes the kinematic viscosity coefficient of the fluid and μ denotes the reciprocal of the magnetic Reynolds number. In this paper, we assume $\nu>0$ and $\mu>0$. Note that the divergence free condition for the magnetic field b is propagated by the second equation in (1.1) if $\nabla\cdot b_0=0$, see [4]. One obvious difference with the usual MHD system is that the Hall-MHD system has the Hall term $\nabla\times((\nabla\times b)\times b)$ due to the happening of the magnetic reconnection when the magnetic shear is large. For the physical background of the magnetic reconnection and the Hall-MHD, we refer the readers to [12,16,17] and references therein.

The Hall-MHD system was derived in a mathematically rigorous way by Acheritogaray, Degond, Frouvelle and Liu [1], where the global existence of weak solutions in the periodic domain was obtained. The global existence of weak solutions in the whole space $\mathbb{R}^3$ and the local well-posedness of classical solution were established by Chae, Degond, and Liu [3]. The authors also obtained a blow-up criterion and the global existence of smooth solution for small initial data. Later, both the blow-up criterion and the small data results were refined by Chae and Lee [4]. In particular, the authors proved that if a regular solution (u,b) on $[0,T)$ satisfies

$$u\in L^q(0,T;L^p(\mathbb{R}^3)) \qquad \text{and} \qquad \nabla b\in L^\gamma(0,T;L^\beta(\mathbb{R}^3)) \tag{1.3}$$

with

$$\frac{3}{p}+\frac{2}{q}\leq 1, \qquad \frac{3}{\beta}+\frac{2}{\gamma}\leq 1 \qquad \text{and} \qquad p,\beta\in(3,\infty] \tag{1.4}$$

then the regular solution can be extended beyond time T. In the limit case $p=\beta=\infty$, it is also shown that if

$$u,\nabla b\in L^2(0,T;BMO(\mathbb{R}^3)) \tag{1.5}$$

then the regular solution can be extended beyond time T, which is an improvement of the Prodi–Serrin condition (1.3)–(1.4).

Partial regularity of weak solutions for the 3D Hall-MHD on plane was studied by Chae and Wolf [7], who proved that the set of possible singularities of a weak solution has the space–time Hausdorff dimension at most two. Optimal temporal decay estimates for weak solutions were obtained by Chae and Schonbek [5]. Energy conservation for weak solutions of the 3D Hall-MHD system was studied by Dumas and Sueur [11]. Local well-posedness of classical solution to the Hall-MHD with fractional magnetic diffusion was obtained by Chae, Wan and Wu [6].

In this paper we will establish a new regularity criterion for the 3D Hall-MHD in term of a Besov norm with restriction only on low frequencies. We adapt the idea from the work of Cheskidov and Shyvdkoy [9] on the regularity problem for the Navier–Stokes equation and Euler's equation. This idea is originated from Kolmogorov's theory of turbulence in [14], which predicts that there is a critical wavenumber above which the viscous term dominates. This method involves some techniques from harmonic analysis, such as the Littlewood–Paley decomposition theory, which are different from classical methods that have been widely used in this area. The