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On the Cauchy problems for polymer flooding with gravitation

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Abstract

We study two systems of conservation laws for polymer flooding in secondary oil recovery, one with gravitation force and one without. For each model, we prove global existence of weak solutions for the Cauchy problems, under rather general assumptions on the flux functions. Approximate solutions are constructed through a front tracking algorithm, and compactness is achieved through the bound on suitably defined wave strengths. As the main technical novelty, we introduce some new nonlinear functionals that yield a uniform bound on the total variation of the flux function.

MSC: 00-01; 99-00

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1. Introduction

In this paper we study two models for enhanced secondary oil recovery by polymer flooding in two phases without adsorption terms. In the first model we neglect the effect of the gravitation force. The model takes the form of a 2×2 system of conservation laws,

$$\begin{cases} s_t + f(s,c)_x = 0\,, \\ (cs)_t + (cf(s,c))_x = 0\,, \end{cases} \tag{1.1}$$

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associated with the initial data

$$\begin{cases} s(0,x) = \bar{s}(x)\,, \\ c(0,x) = \bar{c}(x)\,. \end{cases} \tag{1.2}$$

Here $s \in \mathbb{R}$ is the saturation of the water phase and $c \in \mathbb{R}$ is the fraction of the polymer. For any fixed c, the mapping $s \mapsto f(s,c)$ is typically s-shaped. When c remains constant, the two equations in (1.1) are the same, and the system reduces to a scalar Buckley–Leverett equation [12]. We refer to (1.1)–(1.2) as the *non-gravitation model*.

In the second model, we consider the effect of the gravitation force. The model takes the same form as (1.1), but with a different flux function. We have

$$\begin{cases} s_t + F(s,c)_x = 0\,, \\ (cs)_t + (cF(s,c))_x = 0\,, \end{cases} \tag{1.3}$$

where the flux function $F(s,c)$ is

$$F(s,c) = f(s,c)(1 - K_g \lambda(s,c))\,. \tag{1.4}$$

Here $f(s,c)$ is the same as the fractional flow in (1.1), $K_g \geq 0$ is a constant that reflects the effect of the gravitation force, and $\lambda(s,c)$ denotes the mobility of the oil phase. We referred to (1.3)–(1.2) as the *gravitation model*. For a detailed derivation of the models, see e.g. [27,37,9].

The systems demonstrate many interesting properties. It is observed in [49] that if the initial data $\bar{c}(x)$ is smooth, then $c(t,x)$ remains smooth for all $t > 0$. Such special features can be viewed more conveniently in a Lagrangian coordinate. Following Wagner's construction [50], one can introduce a coordinate change $(t,x) \to (\psi,\phi)$, where (ψ,ϕ) denote the Lagrangian coordinates defined as

$$\begin{cases} \phi_x = -s\,, \qquad \phi_t = f(s,c)\,, \\ \psi = x\,. \end{cases} \tag{1.5}$$

Here ϕ can be viewed as the potential of the first equation in (1.1). The Eulerian system (1.1) now takes an interesting form in the Lagrangian coordinates, for $s > 0$ and $f(s,c) > 0$,

$$\begin{cases} \dfrac{\partial}{\partial\psi}\left(\dfrac{1}{f(s,c)}\right) - \dfrac{\partial}{\partial\phi}\left(\dfrac{s}{f(s,c)}\right) = 0\,, \\ \dfrac{\partial c}{\partial \psi} = 0\,. \end{cases} \tag{1.6}$$

Note that we obtain a triangular system in (1.6), where the second equation is decoupled. Thus the value c remains constant in the time variable ψ. Therefore, if the initial data $c(0,\phi)$ is smooth, it will remain smooth for all $\psi > 0$. Thanks to the classical result of Wagner [50] on the equivalence between the Eulerian and Lagrangian equations, the weak solutions for (1.1) and (1.6) are equivalent for $s \geq s_o > 0$.

For the triangular system (1.6), solutions can be constructed by solving the second equation for c, which is constant in "time" ψ but possibly discontinuous in the "space" ϕ, and plugging it