Available online at www.sciencedirect.com
ScienceDirect
J. Differential Equations 261 (2016) 4338–4367
Journal of Differential Equations
www.elsevier.com/locate/jde

Semicontinuity of attractors for impulsive dynamical systems

E.M. Bonotto [a,*,1], M.C. Bortolan [b], R. Collegari [a,2], R. Czaja [c]

[a] *Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, Campus de São Carlos, Caixa Postal 668, 13560-970 São Carlos, SP, Brazil*
[b] *Departamento de Matemática, Universidade Federal de Santa Catarina, Campus Trindade, 88040-900, Florianópolis, Brazil*
[c] *Institute of Mathematics, University of Silesia, Bankowa 14, 40-007 Katowice, Poland*

Received 13 May 2016; revised 9 June 2016
Available online 1 July 2016

Abstract

In this paper we introduce the concept of collective tube conditions which assures a suitable behaviour for a family of dynamical systems close to impulsive sets. Using the collective tube conditions, we develop the theory of upper and lower semicontinuity of global attractors for a family of impulsive dynamical systems.

MSC: primary 35B41; secondary 34A37, 54H20

Keywords: Impulsive dynamical systems; Global attractors; Semicontinuity

1. Introduction

Perturbations are present in every aspect of the modelling of real world phenomena. Approximate measurements, data collecting, empirical laws and simplifications, for instance, are

* Corresponding author.
E-mail addresses: ebonotto@icmc.usp.br (E.M. Bonotto), mbortolan@gmail.com (M.C. Bortolan), collegari@gmail.com (R. Collegari), rczaja@math.us.edu.pl (R. Czaja).
1 Partially supported by FAPESP 2014/25970-5 and CNPq 307317/2013-7.
2 Partially supported by FAPESP grant 2013/23933-2 and grant 2014/20691-0.

http://dx.doi.org/10.1016/j.jde.2016.06.024
0022-0396/

procedures that introduce small changes in the modelled problem. Such small errors are expected, but they need to be carefully treated. Otherwise, how can we assure that the properties obtained for the model also hold true for the real problem?

To answer this question, we need to study the *continuity* of such problems under small perturbations. We will focus on the following question: what can be said about the asymptotic behaviour of a problem (that is, the behaviour of solutions for large times t) if we make a small perturbation of it?

Even in the case of *continuous dynamical systems*, this question has a very non-trivial answer and the study of the perturbations is divided in the literature, in general, in four steps: the *upper semicontinuity*, the *lower semicontinuity*, the *topological stability* and, lastly, the *geometric stability* (see for instance [1,2,8–11,17,18]). In this paper, we will deal mainly with the upper semicontinuity of impulsive dynamical systems and, also, we shall give some preliminary results on the lower semicontinuity.

We say that a family $\{A_\eta\}_{\eta\in[0,1]}$ of non-empty sets in a metric space (X, d) is **upper semicontinuous** at $\eta = 0$ if

$$\lim_{\eta\to 0} \mathrm{d}_H(A_\eta, A_0) = 0$$

and it is **lower semicontinuous** at $\eta = 0$ if

$$\lim_{\eta\to 0} \mathrm{d}_H(A_0, A_\eta) = 0,$$

where

$$\mathrm{d}_H(A, B) = \sup_{a\in A} \inf_{b\in B} \mathrm{d}(a, b)$$

is the *Hausdorff semidistance* between two non-empty subsets A, B of X.

Roughly speaking, the upper semicontinuity property ensures that the solutions of the perturbed system do not "explode" and follow some solution of the limiting problem. The lower semicontinuity ensures that the solutions of the perturbed system do not "implode" and the perturbed system has, at least, the same degree of complexity that the limiting system.

If one is familiar with the theory of impulsive dynamical systems and their global attractors (see detailed results in [3] and additional results in [4–7,12–16,19,20,22]), there is a natural question to ask: how can we talk about continuity under perturbations of systems which have precisely the *discontinuity* as its main feature?

To answer this question we must remind that, basically, an impulsive dynamical system is formed by a continuous dynamical system and a continuous *impulsive function* (or *jump function*), which gives rise to a discontinuous semiflow, that is, for each initial state, the solution has "jumps" and it is clearly discontinuous. But when we look at *the whole impulsive semiflow*, if the continuous semiflow and the jump function behave continuously under perturbations, there is no reason why the impulsive semiflow would not behave the same. Realizing this, one can see that the study of continuity of impulsive dynamical systems is not a contradictory statement by itself, and involves the study of perturbations of continuous semiflows as well as the study of perturbations of the impulsive functions. This will be the main goal of this work, that is, to study in details the upper semicontinuity of global attractors for impulsive dynamical systems and give a first step towards the study of their lower semicontinuity.