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Traveling wave solutions of Lotka–Volterra competition systems with nonlocal dispersal in periodic habitats

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Abstract

This paper is concerned with space periodic traveling wave solutions of the following Lotka–Volterra competition system with nonlocal dispersal and space periodic dependence,

$$\begin{cases} \frac{\partial u_1}{\partial t} = \int_{\mathbb{R}^N} \kappa(y-x)u_1(t,y)dy - u_1(t,x) + u_1(a_1(x) - b_1(x)u_1 - c_1(x)u_1), & x \in \mathbb{R}^N \\ \frac{\partial u_2}{\partial t} = \int_{\mathbb{R}^N} \kappa(y-x)u_2(t,y)dy - u_2(t,x) + u_2(a_2(x) - b_2(x)u_1 - c_2(x)u_2), & x \in \mathbb{R}^N. \end{cases}$$

Under suitable assumptions, the system admits two semitrivial space periodic equilibria $(u_1^*(x), 0)$ and $(0, u_2^*(x))$, where $(u_1^*(x), 0)$ is linearly and globally stable and $(0, u_2^*(x))$ is linearly unstable with respect to space periodic perturbations. By sub- and supersolution techniques and comparison principals, we show that, for any given $\xi \in S^{N-1}$, there exists a continuous periodic traveling wave solution of the form $(u_1(t,x), u_2(t,x)) = (\Phi_1(x - ct\xi, ct\xi), \Phi_2(x - ct\xi, ct\xi))$ connecting $(u_1^*(\cdot), 0)$ and $(0, u_2^*(\cdot))$ and propagating in the direction of ξ with speed $c > c^*(\xi)$, where $c^*(\xi)$ is the spreading speed of the system in the direction of ξ. Moreover, for $c < c^*(\xi)$ there is no such solution. When the wave speed $c > c^*(\xi)$, we also prove the asymptotic stability and uniqueness of traveling wave solution using squeezing techniques.

MSC: 35B40; 35C07; 45G15; 45M20; 47G20; 92D25

Keywords: Nonlocal dispersal; Lotka–Volterra competition system; Space periodic habitats; Traveling wave solutions; Existence; Asymptotic stability

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1. Introduction

Various forms of dispersal of organisms exist in almost all biological systems [4,13]. Roughly, dispersal means the movement of organisms from one location to another. Random dispersal and nonlocal dispersal are among important forms of dispersal of organisms in population dynamics. The underlying mathematical assumption for random dispersal is that organisms can only move to its immediate surrounding neighborhood and the transition probabilities in all directions are the same. Differential operators, e.g. Δu, are often adopted to model random dispersal (see [3,38,39] for study on the role of dispersal in biological systems). In many biological systems, organisms can travel for some distance and the transition probability from one location to another usually depends upon the distance the organisms traveled. Such dispersal is referred to as nonlocal dispersal and is usually modeled by proper integral operators, such as, $\int_{\mathbb{R}^N}\kappa(y-x)[u(y)-u(x)]dy$ (see [6,17,21,22] for backgrounds on nonlocal dispersal). It is of great importance to study random as well as nonlocal dispersal models arising from applied sciences.

Many random dispersal models arising from biology and ecology, including models for single species and for two competitive species, have been extensively studied and are quite well understood. In recent years, there have also been a lot of studies on nonlocal dispersal models, in particular, models for single species, arising from biology and ecology. However, the understanding of nonlocal dispersal models for two competitive species in spatially and/or temporally heterogeneous environments is still very limited. The objective of the current paper is to study the existence, uniqueness and stability of space periodic traveling wave solutions of the following two species Lotka–Volterra competition system with nonlocal dispersal,

$$\begin{cases} \frac{\partial u_1}{\partial t} = \int_{\mathbb{R}^N}\kappa(y-x)u_1(t,y)dy - u_1(t,x) + u_1(a_1(x)-b_1(x)u_1-c_1(x)u_2), \; x\in\mathbb{R}^N \\ \frac{\partial u_2}{\partial t} = \int_{\mathbb{R}^N}\kappa(y-x)u_2(t,y)dy - u_2(t,x) + u_2(a_2(x)-b_2(x)u_1-c_2(x)u_2), \; x\in\mathbb{R}^N, \end{cases} \tag{1.1}$$

where $u_1=u_1(t,x)$ and $u_2=u_2(t,x)$ denote the densities of two competing species at space location $x\in\mathbb{R}^N$ and time $t\in\mathbb{R}^+$; $\kappa(\cdot)$ is a C^1 nonnegative convolution kernel supported on a ball centered at 0 (that is, $\kappa(z)>0$ if $\|z\|<r_0$ and $\kappa(z)=0$ if $\|z\|\geq r_0$ for some $r_0>0$, where $\|\cdot\|$ denotes the norm in $\mathbb{R}^N$), $\int_{\mathbb{R}^N}\kappa(z)dz=1$; and $a_i(\cdot)$, $b_i(\cdot)$ and $c_i(\cdot)$ $(i=1,2)$ are C^0 and periodic in x with period vector $(p_1,p_2,\cdots,p_N)$ in the sense that $a_i(\cdot+p_l\mathbf{e}_l)=a_i(\cdot)$, $b_i(\cdot+p_l\mathbf{e}_l)=b_i(\cdot)$, $c_i(\cdot+p_l\mathbf{e}_l)=c_i(\cdot)$, $\mathbf{e}_l=(\delta_{l1},\delta_{l2},\ldots,\delta_{lN})$, $\delta_{lk}=1$ if $l=k$ and 0 if $l\neq k$, $l,k=1,2,\ldots,N$, and $b_i(x)$, $c_i(x)>0$ for $x\in\mathbb{R}^N$, while $a_i(\cdot)$ may change sign. In the context of ecology, $a_i(\cdot)$ $(i=1,2)$ are growth rates of the ith species, and $b_i(\cdot)$, $c_i(\cdot)$ account for inter- and intra-competition between the two species.

Observe that, when $u_2(t,x)\equiv 0$, (1.1) reduces to

$$\frac{\partial u_1}{\partial t} = \int_{\mathbb{R}^N}\kappa(y-x)u_1(t,y)dy - u_1(t,x) + u_1(a_1(x)-b_1(x)u_1), \tag{1.2}$$

and when $u_1(t,x)\equiv 0$, (1.1) reduces to

$$\frac{\partial u_2}{\partial t} = \int_{\mathbb{R}^N}\kappa(y-x)u_2(t,y)dy - u_2(t,x) + u_2(a_2(x)-c_2(x)u_2). \tag{1.3}$$