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Entire nodal solutions to the pure critical exponent problem arising from concentration ☆

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Abstract

We obtain new sign changing solutions to the problem

$$(\wp_\infty) \quad -\Delta u = |u|^{2^*-2}u, \quad u \in D^{1,2}(\mathbb{R}^N),$$

for $N \geq 4$ where $2^* := \frac{2N}{N-2}$ is the critical Sobolev exponent. These solutions arise as asymptotic profiles of sign changing solutions to the problem

$$(\wp_p) \quad -\Delta u = |u|^{p-2}u \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

in some bounded smooth domains Ω in $\mathbb{R}^N$ for $p \in (2, 2^*)$ as $p \to 2^*$.

We exhibit solutions u_p to $(\wp_p)$ which blow up at a single point as $p \to 2^*$, developing a peak whose asymptotic profile is a rescaling of a nonradial sign changing solution to problem $(\wp_\infty)$.

We also obtain existence and multiplicity of sign changing nonradial solutions to the Bahri–Coron problem $(\wp_{2^*})$ in annuli.

MSC: primary 35B08, 35B33, 35B40; secondary 35J20, 35J25, 35J61

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1. Introduction

This paper is primarily concerned with the existence of sign changing solutions to the pure critical exponent problem in $\mathbb{R}^N$ which arise as asymptotic profiles in concentration phenomena.

Let $N \geq 3$ and $2^* := \frac{2N}{N-2}$ be the critical Sobolev exponent. It is well-known that, up to translations and dilations, problem

$$(\wp_\infty) \qquad \begin{cases} -\Delta u = |u|^{2^*-2}u, \\ u \in D^{1,2}(\mathbb{R}^N), \end{cases}$$

has a unique positive solution, called the standard bubble, given by

$$U(x) = [N(N-2)]^{\frac{N-2}{4}} \frac{1}{(1+|x|^2)^{\frac{N-2}{2}}}.$$

W. Ding showed that $(\wp_\infty)$ has infinitely many sign changing solutions which are invariant under the conformal action of the group $\mathcal{G} := O(k) \times O(m)$ on $\mathbb{R}^N$, induced by the orthogonal action of $\mathcal{G}$ on the N-dimensional sphere $\mathbb{S}^N$ via the stereographic projection, if $k+m = N+1$, $m, k \geq 2$ [13]. The Sobolev embedding, when restricted to the space of $\mathcal{G}$-invariant functions, turns out to be compact, so standard variational methods yield this result.

Recently, del Pino, Musso, Pacard and Pistoia constructed other sign changing solutions using the Lyapunov–Schmidt reduction method [11]. Their solutions look like a sum of a positive standard bubble centered at the origin and k negative standard bubbles, conveniently rescaled and evenly distributed along a circle.

In this paper we obtain sign changing solutions to $(\wp_\infty)$ which are different from those in [13,11]. Our solutions occur as asymptotic profiles, as $p \to 2^*$, of solutions to the problem

$$(\wp_p) \qquad \begin{cases} -\Delta u = |u|^{p-2}u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$, $N \geq 3$ and $p \in (2, 2^*)$.

Bahri, Li and Rey observed in [2] that positive solutions to $(\wp_p)$ either converge to a positive solution of the critical problem $(\wp_{2^*})$ or blow up at a finite number of points, as $p \to 2^*$, developing a peak at each of these points whose asymptotic profile is a rescaling of the standard bubble. Existence and nonexistence of positive solutions with multiple blow-up points have been studied in [15,21].

Standard bubbles have been used to produce sign changing solutions to $(\wp_p)$ which blow up at multiple points or which develop multiple blow-up at a single point as $p \to 2^*$, see, e.g., [3–5,19, 20]. In all cases, the asymptotic profile of the solutions is a sum of positive and negative bubbles blowing up at different points or at the same point with different velocities. Concentration of sign changing solutions to problem $(\wp_p)$ in dimension $N = 2$ was recently addressed in [12], where sign changing solutions are exhibited whose limit profile, as $p \to \infty$, is a superposition of two bubbles, a regular and a singular solution of the Liouville problem in $\mathbb{R}^2$, with opposite signs.