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Weak stability of the plasma–vacuum interface problem

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Abstract

We consider the free boundary problem for the two-dimensional plasma–vacuum interface in ideal compressible magnetohydrodynamics (MHD). In the plasma region, the flow is governed by the usual compressible MHD equations, while in the vacuum region we consider the Maxwell system for the electric and the magnetic fields. At the free interface, driven by the plasma velocity, the total pressure is continuous and the magnetic field on both sides is tangent to the boundary.

We study the linear stability of rectilinear plasma–vacuum interfaces by computing the Kreiss–Lopatinskiĭ determinant of an associated linearized boundary value problem. Apart from possible resonances, we obtain that the piecewise constant plasma–vacuum interfaces are always weakly linearly stable, independently of the size of tangential velocity, magnetic and electric fields on both sides of the characteristic discontinuity.

We also prove that solutions to the linearized problem obey an energy estimate with a loss of regularity with respect to the source terms, both in the interior domain and on the boundary, due to the failure of the uniform Kreiss–Lopatinskiĭ condition, as the Kreiss–Lopatinskiĭ determinant associated with this linearized boundary value problem has roots on the boundary of the frequency space. In the proof of the a priori estimates, a crucial part is played by the construction of symmetrizers for a reduced differential system, which has poles at which the Kreiss–Lopatinskiĭ condition may fail simultaneously.

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1. Introduction

Plasma–vacuum interface problems appear in the mathematical modeling of plasma confinement by magnetic fields in thermonuclear energy production (as in Tokamaks, Stellarators; see, e.g., [1]). There are also important applications in astrophysics, where the plasma–vacuum interface problem can be used for modeling the motion of a star or the solar corona when magnetic fields are taken into account.

In [2,3], the authors obtained the local-in-time existence and uniqueness of solutions to the free boundary problem for the plasma–vacuum interface in ideal compressible magnetohydrodynamics (MHD), by considering the *pre-Maxwell dynamics* for the magnetic field in the vacuum region, as usually assumed in the classical formulation. The linearized stability of the relativistic case has been addressed by Trakhinin in [4], in the case of plasma expansion in vacuum. The paper [5] is devoted to the study of the linearized stability for the non-relativistic case, but, instead of the pre-Maxwell dynamics, in the vacuum region the displacement current was taken into account and the complete system of *Maxwell equations* for the electric and the magnetic fields was considered. The introduction of this model aims at investigating the influence of the electric field in vacuum on the well-posedness of the problem, as in the classical pre-Maxwell dynamics such an influence is hidden. See also [6] for a similar problem.

For the relativistic plasma–vacuum problem, Trakhinin [4] has shown the possible ill-posedness in the presence of a sufficiently strong vacuum electric field. Since relativistic effects play a rather passive role in the analysis of [4], it is natural to expect the same for the non-relativistic problem. On the contrary, in [5] it was shown that a *sufficiently weak* vacuum electric field precludes ill-posedness and gives the well-posedness of the linearized problem, thus somehow justifying the practice of neglecting the displacement current in the classical pre-Maxwell formulation when the vacuum electric field is weak enough.

In all the previously cited papers [5,6,2–4] the analysis is performed under a suitable stability condition stating that at each point of the free interface the magnetic fields on both sides are not parallel, see also [7–11] for the similar condition on current-vortex sheets. These works show that non-parallel magnetic fields may stabilize the motion. The main technical reason of why the stabilization occurs is that the non-collinearity of the magnetic fields is a sufficient condition for the ellipticity of the symbol of the boundary operator, namely the operator that is obtained from the boundary conditions and applies to the function describing the free interface, and this gives a control of the space-time gradient of such a function.

On the other hand, one could guess that the well-posedness could be guaranteed as well for problems without ellipticity of the boundary operator, and necessarily with less regularity of the free interface, provided a suitable stability condition is assumed.

In this regard, in the recent paper [12] Y. Trakhinin considered the three dimensional plasma–vacuum interface problem in the classical pre-Maxwell dynamics formulation with non-elliptic interface symbol. In [12] a basic L^2 a priori estimate was derived for the linearized problem with variable coefficients, in the case that the unperturbed plasma and vacuum magnetic fields are everywhere parallel on the interface, provided a Rayleigh–Taylor sign condition on the jump of the normal derivative of the total pressure is satisfied at each point of the interface. The general