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Optimal decay rate for the local energy of a unbounded network

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Abstract

We consider the wave equation on a unbounded network with Dirichlet and Kirchhoff conditions. We study the local energy decay of the solution. We prove that the energy is exponentially decaying and we give an exact formula for exponential decay rate. The limit energy is also given in terms of the initial conditions. The results are obtained using two approaches. A direct one uses type τ operators in the case of equal edge lengths. The other one is based on a spectral investigation of an associated linear operator leading to the correspondence between the resonances width and the local energy decay rate.

MSC: primary 35B37, 35B40, 35L05; secondary 35B35

Keywords: Network of strings; Local energy decay; Energy estimate; D'Alembert formula; Resolvent; Resonances

Contents

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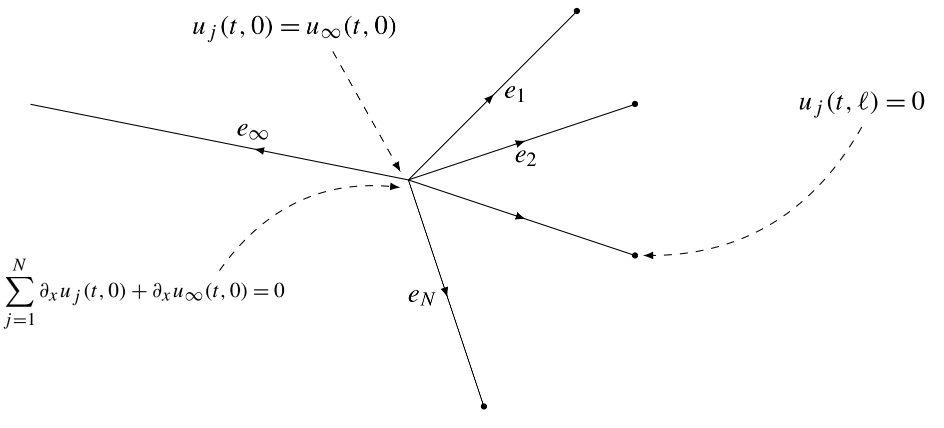
1. Introduction

We consider a network of mixed strings, bounded and unbounded. This is a unbounded structure Γ with a finite number N of compact edges e_j, $j = 1, \cdots, N$ of the same lengths $\ell > 0$ to which we attach one or more copies of the interval $[0, \infty[$. If we consider the case of a network with N edges e_j, $j = 1, \ldots, N$ and a single infinite edge e_∞, all of them connected at a single vertex, then we may identify each finite edge to $[0, \ell]$ and the infinite one to the interval $[0, +\infty[$. Thus, the network model is as shown in the following picture:



In the past decades, many researchers analyzed models of vibrating networks. Some of them tried to solve the controllability and stabilization problems [1–6] and others were interested in the spectral study of this kind of structures [7,5,8,9], but a lot remains to be done in order to have a complete theory. The main novelty is that this work concerns the local energy decay in unbounded 1-d flexible multi-structures. More precisely, we study the local energy decay of the solution of the wave equation with some boundary and initial conditions on the network Γ described below. The local energy decay has been widely considered on domains of $\mathbb{R}^n$. For this, Vainberg [10,11] (see also N. Burq [12] and G. Vodev [13]), used the estimates on the cutoff resolvent in order to find results on the local energy decay for an exterior domain of conservative wave equation. They proved that the decay rate depends on the geometry of the domain. Their results was extended by L. Aloui and M. Khenissi [14] and by M. Khenissi [15] to the damping wave equation under the exterior geometric condition (EGC) on the action region of the damping term [14].