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Uniform and strict persistence in monotone skew-product semiflows with applications to non-autonomous Nicholson systems ☆

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Abstract

We determine sufficient conditions for uniform and strict persistence in the case of skew-product semiflows generated by solutions of non-autonomous families of cooperative systems of ODEs or delay FDEs in terms of the principal spectrums of some associated linear skew-product semiflows which admit a continuous separation. Our conditions are also necessary in the linear case. We apply our results to a noncooperative almost periodic Nicholson system with a patch structure, whose persistence turns out to be equivalent to the persistence of the linearized system along the null solution.

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1. Introduction

Different notions of persistence have been introduced and investigated in the mathematical theory of dynamical systems. Basically all of them mean that in the long run the trajectories of the system place themselves above a prescribed region of the phase space. In many practical applications this region is determined by a constant or null value of the state variables. In this last case persistence means that the solutions eventually become uniformly strongly positive. The dynamical theory of persistence has been extensively used in biological population dynamics, ecology, epidemiology, chemical reactions, game theory, neural networks and other important areas of applied sciences and engineering. The references Anderson [2], Bonneuil [3], Butler and Wolkowicz [4], Calzada et al. [5], Cantrell and Cosner [6], Craciun et al. [7], Hale and Waltman [19], Hofbauer and Sigmund [22], Johnston et al. [25], Smith and Thieme [49] and Takeuchi [50] illustrate some of the classical and also more recent applications of the mentioned theory in these fields.

The presence and consequences of persistence in dynamical systems have been broadly investigated in the literature using topological methods, Lypunov functions, comparison methods, Morse decompositions, invariant splitting, Lyapunov exponents and computational methods, among other techniques. The papers by Faria and Röst [12], Freedman and Ruan [15], Garay and Hofbauer [16], Hetzer and Shen [20], Hirsch et al. [21], Langa et al. [26], Magal and Zhao [28], Mierczyński and Shen [30], Mierczyński et al. [31], Novo et al. [36], Salceanu and Smith [43], Schreiber [44,45], Thieme [51,52], Wang and Zhao [54], and references therein, provide a long but not complete list of works on this topic.

The objective of this paper is to continue the study of the dynamical theory of persistence in monotone skew-product semiflows generated by non-autonomous differential equations, initiated in Novo et al. [36], as well as to show the applicability of this theory in the description of some mathematical models widely investigated in the literature, which are locally cooperative in small regions of the phase space. In particular we give a complete characterization of persistence for non-autonomous n-dimensional Nicholson systems with a patchy structure, which are able to model the temporal changes of the environment.

We investigate relevant properties of non-autonomous dynamical systems by using the skew-product formalism. The phase space is a product space $\Omega \times X$, where the base Ω is a compact metric space under the action of a continuous flow $\sigma : \mathbb{R} \times \Omega \to \Omega$, $(t, \omega) \mapsto \omega{\cdot}t$, and the state space X is a strongly ordered Banach space with a normal positive cone. The skew-product semiflow is defined by $\tau : \mathbb{R}_+ \times \Omega \times X \to \Omega \times X$, $(t, \omega, x) \mapsto (\omega{\cdot}t, u(t, \omega, x))$, where the map u satisfies the usual semicocycle identity, it is monotone on some region of the phase space and smooth with respect to the state component x. We frequently assume that the base flow is minimal, *i.e.*, that all the trajectories on Ω are dense. In particular, this formalism permits to carry out a dynamical study of solutions of non-autonomous differential equations in which the temporal variation of the vector field is almost periodic, almost automorphic, or more in general, recurrent. The papers by Chow and Leiva [8,9], Johnson et al. [23], Johnson et al. [24], Sacker and Sell [41,42], Novo and Obaya [34], Novo et al. [35], Poláčik and Tereščák [39], Shen and Yi [47] and Yi [55] contain the main mathematical ingredients required to follow the contents of this work.

We introduce natural definitions of uniform and strict persistence which become relevant in applications. Both definitions agree with the concept of uniform (strong) ρ-persistence in the terms of Smith and Thieme [49] for an adequate choice of the map $\rho : X \to \mathbb{R}_+$. We develop part of the methods mentioned above to investigate the presence of persistence in non-autonomous