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Entire solutions of nonlocal dispersal equations with monostable nonlinearity in space periodic habitats

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Abstract

This paper is concerned with the new types of entire solutions other than traveling wave solutions of nonlocal dispersal equations with monostable nonlinearity in space periodic habitats. We first establish the existence and properties of spatially periodic solutions connecting two steady states. Then new types of entire solutions are constructed by combining the rightward and leftward pulsating traveling fronts with different speeds and a spatially periodic solution. Finally, for a class of special heterogeneous reaction, we further establish the uniqueness of entire solutions and the continuous dependence of such an entire solution on parameters, such as wave speeds and the shifted variables. In other words, we build a five-dimensional manifold of solutions and the traveling wave solutions are on the boundary of the manifold.

MSC: 35K57; 35R20; 92D25

Keywords: Nonlocal dispersal; Monostable nonlinearity; Spatially periodic; Continuous dependence; Entire solution

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1. Introduction and preliminary

The current paper deals with entire solutions of the following spatially periodic monostable equation with nonlocal dispersal:

$$u_t(x,t)=\int\limits_{\mathbb{R}^N} J(y-x)u(y,t)\mathrm{d}y-u(x,t)+u(x,t)f(x,u(x,t)),\ x\in\mathbb{R}^N, \tag{1.1}$$

where $u(x,t)$ denotes the population density of species at space x and time t. $\int_{\mathbb{R}^N} J(y-x)u(y,t)\mathrm{d}y-u(x,t)$ is called the nonlocal dispersal and represents transportation due to long range dispersion mechanisms. Particularly, $J(\cdot)$ is a C^1 symmetric convolution kernel supported on a ball centered at the origin (that is, there is a $\delta_0>0$ such that $J(z)>0$ if $\|z\|<\delta_0$, $J(z)=0$ if $\|z\|\ge\delta_0$, where $\|\cdot\|$ denotes the norm in $\mathbb{R}^N$ and δ_0 represents the nonlocal dispersal distance), and satisfies $\int_{\mathbb{R}^N} J(z)\mathrm{d}z=1$. The nonlinear term $uf(x,u)$ satisfies certain monostable assumptions and $f(x,u)$ is periodic in x with period vector $\mathbf{p}=(p_1,p_2,...,p_N)$ (that is, $f(\cdot+p_i\mathbf{e_i},\cdot)=f(\cdot,\cdot)$, $\mathbf{e_i}=(\delta_{i1},\delta_{i2},...,\delta_{iN})$, $\delta_{ij}=1$ if $i=j$ and $\delta_{ij}=0$ if $i\neq j$, $i,j=1,2,...,N$). Here, the periodic dependence of f with respect to x reflects the effects of nonhomogeneous environment on the population growth.

Monostable equations have been widely applied in modeling population dynamics in biology and ecology. It is shown that the movements and interactions of all individuals described in equation (1.1) are free and large-range, and we call this phenomenon nonlocal dispersal. Correspondingly, when the individuals move randomly between the adjacent spatial locations in the continuous space, we can obtain the following classical reaction-diffusion equation:

$$u_t(x,t)=\triangle u(x,t)+u(x,t)f(x,u(x,t)),\ x\in\mathbb{R}^N. \tag{1.2}$$

Moreover, if the species live in a patchy environment, we get the corresponding spatially discrete form of (1.2):

$$\begin{aligned}u_t(j,t)=&\,d_{j+1}u(j+1,t)+d_ju(j-1,t)-(d_{j+1}+d_j)u(j,t)\\&+u(j,t)f(j,u(j,t)),\ j\in\mathbb{Z}.\end{aligned} \tag{1.3}$$

As we know, spatial spread and front propagation dynamics (for example, traveling wave solutions and spreading speeds) are some important dynamical issues about (1.1), (1.2) and (1.3). In addition to traveling wave solutions and spreading speeds, another important issue in biology and ecology is the interaction between traveling wave solutions. Mathematically, this phenomenon can be described by a class of *entire solutions* which are defined for whole space and all time. In fact, traveling wave solutions are only special examples of the so-called entire solutions. The study on entire solutions is crucial and significant. Simply speaking, entire solutions provide some new spread and invasion ways of the epidemic and species [27,35,47]. Furthermore, entire solutions can help us for the mathematical understanding of transient dynamics and the structures of the global attractor [34].

Since the pioneering works by Fisher [10] and Kolmogorov et al. [22] on the special case of (1.2) with $f(x,u)=1-u$, equation (1.2) and some other versions of it have been extensively studied during the past decades, one can refer to [1–3,11,16,29,28,36,33,44,45] for the study of traveling wave solutions and spreading speeds. Likewise, many researchers have devoted to the