Available online at www.sciencedirect.com
ScienceDirect
J. Differential Equations 261 (2016) 821–889
Journal of Differential Equations
www.elsevier.com/locate/jde

The free boundary Euler equations with large surface tension

Marcelo M. Disconzi [a,1], David G. Ebin [b,*]

[a] Department of Mathematics, Vanderbilt University, Nashville, TN, USA
[b] Department of Mathematics, Stony Brook University, Stony Brook, NY, USA

Received 16 February 2016
Available online 31 March 2016

Abstract

We study the free boundary Euler equations with surface tension in three spatial dimensions, showing that the equations are well-posed if the coefficient of surface tension is positive. Then we prove that under natural assumptions, the solutions of the free boundary motion converge to solutions of the Euler equations in a domain with fixed boundary when the coefficient of surface tension tends to infinity.

Keywords: Large surface tension; Free boundary; Euler equations; Incompressible fluid

Contents

* Corresponding author.
E-mail addresses: marcelo.disconzi@vanderbilt.edu (M.M. Disconzi), ebin@math.sunysb.edu (D.G. Ebin).
[1] Marcelo M. Disconzi is partially supported by NSF grant 1305705.

http://dx.doi.org/10.1016/j.jde.2016.03.029
0022-0396/

1. Introduction

Consider the initial value problem for the motion of an incompressible inviscid fluid with free boundary whose equations of motion are given in Lagrangian coordinates by (see below for the equations in Eulerian coordinates)

$$\begin{cases} \ddot{\eta} = -\nabla p \circ \eta & \text{in } \Omega, & \text{(a)} \\ \operatorname{div}(u) = 0 & \text{in } \eta(\Omega), & \text{(b)} \\ p|_{\partial\eta(\Omega)} = \kappa\mathcal{A} & \text{on } \partial\eta(\Omega), & \text{(c)} \\ \eta(0) = \operatorname{id}, \ \dot{\eta}(0) = u_0, & & \text{(d)} \end{cases} \tag{1.1}$$

where Ω is a domain in $\mathbb{R}^n$; $\eta(t,\cdot)$ is, for each t, a volume preserving embedding $\eta(t) : \Omega \to \mathbb{R}^n$ representing the fluid motion, with t thought of as the time variable ($\eta(t,x)$ is the position at time t of the fluid particle that at time zero was at x); "$\dot{}$" denotes derivative with respect to t; $\Omega(t) = \eta(t)(\Omega)$; $u : \Omega(t) \to \mathbb{R}^n$ is a vector field on $\Omega(t)$ defined by $u = \dot{\eta} \circ \eta^{-1}$ (it represents the fluid velocity); $\mathcal{A}$ is the mean curvature of the boundary of the domain $\Omega(t)$; p is a real valued function on $\Omega(t)$ called the pressure; finally, κ is a non-negative constant known as the coefficient of surface tension. id denotes the identity map, u_0 is a given divergence free vector field on Ω, and div means divergence. The unknowns are the fluid motion η and the pressure p, but notice that the system (1.1) is coupled in a non-trivial fashion in the sense that the other quantities appearing in (1.1), namely u, $\mathcal{A}$, and $\Omega(t)$, depend explicitly or implicitly on η and p.

With suitable assumptions, we shall prove the following result, concerning the existence of solutions to (1.1) and the behavior of solutions when the coefficient of surface tension is large, i.e., in the limit $\kappa \to \infty$. A precise statement is given in Theorem 1.2 below.

Theorem *(Main Result). (See Theorem 1.2 for precise statements.) Under appropriate conditions on the initial condition u_0 and on $\partial\Omega$, we have:*

1) *If $\kappa > 0$, then (1.1) is well posed.*