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Hydrodynamic limit with geometric correction of stationary Boltzmann equation

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Abstract

We consider the hydrodynamic limit of a stationary Boltzmann equation in a unit plate with in-flow boundary. The classical theory claims that the solution can be approximated by the sum of interior solution which satisfies steady incompressible Navier–Stokes–Fourier system, and boundary layer derived from Milne problem. In this paper, we construct counterexamples to disprove such formulation in L^∞ both for its proof and result. Also, we show the hydrodynamic limit with a different boundary layer expansion with geometric correction.

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Keywords: Normal singularity; Boundary layer; Geometric correction; Boussinesq relation

Contents

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1. Introduction

1.1. Problem formulation

We consider stationary Boltzmann equation for distribution density $F^\epsilon(\vec{x},\vec{v})$ in a two-dimensional unit plate $\Omega=\{\vec{x}=(x_1,x_2):\ |\vec{x}|\leq 1\}$ with velocity $\Sigma=\{\vec{v}=(v_1,v_2)\in\mathbb{R}^2\}$ as

$$\begin{cases} \epsilon\vec{v}\cdot\nabla_x F^\epsilon &= Q[F^\epsilon,F^\epsilon] \ \text{ in } \ \Omega\times\mathbb{R}^2,\\ F^\epsilon(\vec{x}_0,\vec{v}) &= B^\epsilon(\vec{x}_0,\vec{v}) \ \text{ for } \ \vec{x}_0\in\partial\Omega \ \text{ and } \ \vec{n}(\vec{x}_0)\cdot\vec{v}<0, \end{cases} \tag{1.1}$$

where $\vec{n}(\vec{x}_0)$ is the outward normal vector at $\vec{x}_0$ and the Knudsen number ϵ satisfies $0<\epsilon<<1$. Here we have

$$Q[F,G]=\int_{\mathbb{R}^2}\int_{\mathcal{S}^1} q(\vec{\omega},\left|\vec{\mathrm{u}}-\vec{v}\right|)\bigg(F(\vec{\mathrm{u}}_*)G(\vec{v}_*)-F(\vec{\mathrm{u}})G(\vec{v})\bigg)\mathrm{d}\vec{\omega}\mathrm{d}\vec{\mathrm{u}}, \tag{1.2}$$

with

$$\vec{\mathrm{u}}_*=\vec{\mathrm{u}}+\vec{\omega}\bigg((\vec{v}-\vec{\mathrm{u}})\cdot\vec{\omega}\bigg), \qquad \vec{v}_*=\vec{v}-\vec{\omega}\bigg((\vec{v}-\vec{\mathrm{u}})\cdot\vec{\omega}\bigg), \tag{1.3}$$

and the hard-sphere collision kernel

$$q(\vec{\omega},\left|\vec{\mathrm{u}}-\vec{v}\right|)=q_0\left|\vec{\mathrm{u}}-\vec{v}\right|\left|\cos\phi\right|, \tag{1.4}$$