



# Navier wall law for nonstationary viscous incompressible flows

Mitsuo Higaki

*Mathematical Institute, Tohoku University, Sendai 980-8578, Japan*

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## Abstract

We study the Navier wall law for the two-dimensional initial boundary value problem of the Navier–Stokes equations in a domain with a rough boundary. The Navier wall law is verified for the initial data in  $C^1$  class under the natural compatibility condition. Our proof relies on the boundary layer analysis and the  $L^\infty$  theory of the Navier–Stokes equations in the half space.

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## 1. Introduction

To understand the effect of rough boundaries against the behavior of fluid flows is one of the important themes in fluid mechanics. Indeed, this problem arises in many real applications, such as the flows on surfaces with fine riblets. For an overview, we refer to Mikelić [16].

In order to describe this situation mathematically, we introduce the two-dimensional Navier–Stokes system in a domain with a rough boundary  $\Omega^\varepsilon = \{(x_1, x_2) \in \mathbb{R} \times \mathbb{R}; \varepsilon\omega(x_1/\varepsilon) < x_2 < \infty\}$

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*E-mail address:* [sb4m21@math.tohoku.ac.jp](mailto:sb4m21@math.tohoku.ac.jp).

$$\left\{ \begin{array}{l} \partial_t u^\varepsilon - \Delta u^\varepsilon + u^\varepsilon \cdot \nabla u^\varepsilon + \nabla p^\varepsilon = 0, \quad t > 0, \quad x \in \Omega^\varepsilon, \\ \nabla \cdot u^\varepsilon = 0, \quad t \geq 0, \quad x \in \Omega^\varepsilon, \\ u^\varepsilon(x_1, x_2) \text{ is } 2\pi\text{-periodic in } x_1, \quad t \geq 0, \\ u^\varepsilon|_{\partial\Omega^\varepsilon} = 0, \quad t > 0, \\ u^\varepsilon|_{t=0} = u_0, \quad x \in \Omega^\varepsilon. \end{array} \right. \quad (\text{NS}^\varepsilon)$$

The unknown functions  $u^\varepsilon = (u_1^\varepsilon, u_2^\varepsilon)$  and  $p^\varepsilon$  are respectively the velocity field and the pressure field of the fluid and  $u_0 = (u_{0,1}, u_{0,2})$  is an initial velocity field. Throughout the present paper we use the standard notations for differential operators, such as  $\nabla \cdot u = \partial_1 u_1 + \partial_2 u_2$ . The boundary function  $\omega : \mathbb{R} \rightarrow (-1, -1/2)$  is assumed to be smooth and  $2\pi$ -periodic. The parameter  $\varepsilon = 1/N$ ,  $N \in \mathbb{N}$ , characterizes the amplitude and the pulse width (namely, the roughness) of the boundary  $\partial\Omega^\varepsilon$ . In this setting we consider the asymptotic profile of the flow  $u^\varepsilon$  as  $\varepsilon \rightarrow 0$ .

A typical approach to describe the averaged effect of such an irregular boundary on  $u^\varepsilon$  is to replace the actual rough boundary by an artificially smooth one, but instead, the boundary condition on the new boundary is imposed so as to reflect the effect of the roughness of the original boundary. In our mathematical setting, this process corresponds to consider the Navier–Stokes system in the half-space  $\mathbb{R}_+^2 = \{x \in \mathbb{R}^2; x_2 > 0\}$

$$\left\{ \begin{array}{l} \partial_t u - \Delta u + u \cdot \nabla u + \nabla p = 0, \quad t > 0, \quad x \in \mathbb{R}_+^2, \\ \nabla \cdot u = 0, \quad t \geq 0, \quad x \in \mathbb{R}_+^2, \\ u(x_1, x_2) \text{ is } 2\pi\text{-periodic in } x_1, \quad t \geq 0, \\ u|_{t=0} = a, \quad x \in \mathbb{R}_+^2, \end{array} \right. \quad (\text{NS})$$

with a new boundary condition on  $\partial\mathbb{R}_+^2$  which should reflect the averaged effect of the rough boundary. An immediate example is the classical no-slip boundary condition:

$$u = 0 \quad \text{on } \partial\mathbb{R}_+^2, \quad (\text{Di})$$

although it does not take the behavior of  $u^\varepsilon$  near the boundary into account. The new boundary conditions derived through the above process are called “wall laws”, and there is a lot of literature on the formal derivation in various settings. However, the derivation of wall laws often relies on formal computations and it is therefore important to justify the wall laws with a mathematical rigor. For the formal derivations of wall laws and its numerical validations, we refer to Achdou, Mohammadi, Pironneau, and Valentin [2] and Achdou, Pironneau, and Valentin [3].

So far the justification of wall laws is discussed mathematically mainly for the stationary viscous incompressible flows subject to the no-slip boundary condition. In the pioneering work of Jäger and Mikelić [13], the mathematical justification is given when the two-dimensional stationary channel flows are close to the small Poiseuille flow  $u^0$ . This result is extended for random rough boundaries and almost periodic boundaries by Basson and Gérard-Varet [6] and by Gérard-Varet and Masmoudi [12], respectively; see Dalibard and Gérard-Varet [9] for further generalization. In the papers mentioned above, the derivation of the wall law relies on the following formal expansion

$$u^\varepsilon \sim u^0 + \varepsilon \frac{\partial u_1^0}{\partial x_2} \Big|_{x_2=0} v^0 \left( \frac{x}{\varepsilon} \right),$$

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