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On exponential stability of gravity driven viscoelastic flows

Fei Jiang [a,*], Guochun Wu [b], Xin Zhong [c]

[a] *College of Mathematics and Computer Science, Fuzhou University, Fuzhou 350108, China*
[b] *Department of Mathematics, Putian University, Putian 351100, China*
[c] *Institute of Applied Mathematics, AMSS, Chinese Academy of Sciences, Beijing 100190, China*

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Abstract

We investigate stability of an equilibrium state to a nonhomogeneous incompressible viscoelastic fluid driven by gravity in a bounded domain $\Omega \subset \mathbb{R}^3$ of class C^3. First, we establish a critical number κ_C, which depends on the equilibrium density and the gravitational constant, and is a threshold of the elasticity coefficient κ for instability and stability of the linearized perturbation problem around the equilibrium state. Then we prove that the equilibrium state is exponential stability provided that $\kappa > \kappa_C$ and the initial disturbance quantities around the equilibrium state satisfy some relations. In particular, if the equilibrium density $\bar{\rho}$ is a Rayleigh–Taylor (RT) type and $\bar{\rho}'$ is a constant, our result strictly shows that the sufficiently large elasticity coefficient can prevent the RT instability from occurrence.

Keywords: Viscoelastic fluid; Equilibrium state; Rayleigh–Taylor instability; Stability

1. Introduction

The models of viscoelastic fluids formulated by Oldroyd [36,38] (see [2,37,42] for alternative derivations and perspectives), especially the Oldroyd-B model, have been studied by many au-

* Corresponding author.
E-mail addresses: jiangfei0591@163.com (F. Jiang), guochunwu@126.com (G. Wu), xzhong1014@amss.ac.cn (X. Zhong).

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thors (see [17,29,34] and the references cited therein) since the pioneering work of Renardy [43] and Guillopé and Saut (see [10] and the references cited therein). In this article, we consider the following nonhomogeneous incompressible idea Oldroyd-B model in the presence of a uniform gravitational field in a bounded domain $\Omega \subset \mathbb{R}^3$:

$$\begin{cases} \rho_t + v \cdot \nabla \rho = 0, \\ \rho v_t + \rho v \cdot \nabla v - \mu \Delta v - \kappa \operatorname{div}(\rho U U^\top) + \nabla p = -g\rho e_3, \\ U_t + v \cdot \nabla U - \nabla v U = 0, \\ \operatorname{div} v = 0. \end{cases} \tag{1.1}$$

Here $\rho := \rho(x,t)$, $v := v(x,t)$ and $p := p(x,t)$ are the density, velocity, and pressure of fluid, respectively. $U(x,t)$ denotes the deformation tensor (a 3×3 matrix-valued function). The known physical parameter μ and κ are positive, and denote the viscosity coefficient and elasticity coefficient of fluid, respectively. $g > 0$ stands for the gravitational constant, $e_3 = (0,0,1)^\top$ for the vertical unit vector, and $-ge_3$ for the gravitational force.

There is huge literature on the studies about global existence and behaviors of solutions to (1.1) in the case that the density ρ is constant, refer to [30–33] and references therein. Despite the progress on well-posedness for idea Oldroyd-B model, to the best of our knowledge, there is no an available result concerning the influence of elasticity coefficient κ on the system (1.1). As is known, viscoelasticity is the property of materials that exhibit both viscous and elastic characteristics when undergoing deformation. In particular, elastic fluids strain when stretched and quickly return to their original state once the stress is removed. Hence, it is of great interest to study the influence of elasticity coefficient on the perturbation problem of (1.1). To this end, we consider an equilibrium state to the system (1.1):

$$v(t,x) = 0, \ U = I, \ \nabla(\bar{p} + \kappa \bar{\rho}) = -\bar{\rho} g e_3 \text{ in } \Omega, \tag{1.2}$$

where I is an identity matrix and $\bar{\rho}$ satisfies

$$\bar{\rho} \in C^1(\overline{\Omega}) \text{ and } \inf_{x \in \overline{\Omega}} \bar{\rho} > 0. \tag{1.3}$$

It is not hard to show that the equilibrium density $\bar{\rho}$ depends only on x_3, the third component of x. Hence we denote $\bar{\rho}' := \partial_{x_3} \bar{\rho}$ in this paper. Moreover, we can compute out the corresponding steady pressure $\bar{p}$ determined by $\bar{\rho}$.

Now, denote the perturbation quantities by

$$\varrho = \rho - \bar{\rho}, \quad u = v - 0, \quad q = p - \bar{p}, \quad V = U - I,$$

then the system (1.1) can be written as the perturbation equations:

$$\begin{cases} \varrho_t + u \cdot \nabla(\bar{\rho} + \varrho) = 0, \\ (\bar{\rho} + \varrho) u_t + (\bar{\rho} + \varrho) u \cdot \nabla u + \nabla q_1 \\ \qquad = \mu \Delta u + \kappa \operatorname{div}((\bar{\rho} + \varrho)(S(V) + V V^\top) - g \varrho e_3, \\ V_t + u \cdot \nabla V - \nabla u (I + V) = 0, \\ \operatorname{div} u = 0, \end{cases} \tag{1.4}$$