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ScienceDirect

J. Differential Equations 260 (2016) 7861–7891

Journal of Differential Equations

www.elsevier.com/locate/jde

On massless electron limit for a multispecies kinetic system with external magnetic field

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Received 24 September 2015; revised 4 February 2016

Available online 18 February 2016

Abstract

We consider a three-dimensional kinetic model for a two species plasma consisting of electrons and ions confined by an external nonconstant magnetic field. Then we derive a kinetic-fluid model when the mass ratio m_e/m_i tends to zero.

Each species initially obeys a Vlasov-type equation and the electrostatic coupling follows from a Poisson equation. In our modeling, ions are assumed non-collisional while a Fokker–Planck collision operator is taken into account in the electron equation. As the mass ratio tends to zero we show convergence to a new system where the macroscopic electron density satisfies an anisotropic drift-diffusion equation. To achieve this task, we overcome some specific technical issues of our model such as the strong effect of the magnetic field on electrons and the lack of regularity at the limit. With methods including renormalized solutions, relative entropy dissipation and velocity averages, we establish the rigorous derivation of the limit model.

MSC: 35Q83; 35Q84; 35B25; 82D10; 76X05

Keywords: Multispecies; Plasma physics; Magnetic field; Vlasov–Poisson; Vlasov–Poisson–Fokker–Planck; Drift-diffusion

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http://dx.doi.org/10.1016/j.jde.2016.02.005

1. Introduction

In many plasma physics applications, the numerical simulation of full multi-species kinetic systems of equations can be extremely expensive in computer time. Indeed, since the typical time, space and velocity scales of each species differ from several orders of magnitude, it requires a fine discretization to accurately approximate the different scales. We refer to [2] for a discussion on these issues in the two-species Vlasov–Poisson case. Therefore, part of the problem is sometimes overcome by making simplifying assumptions on species with negligible contribution to the whole dynamic. In this paper, we are interested in reducing a kinetic model by taking the limit when the mass ratio between light and heavy particles, namely electrons and ions, tends to 0.

The charged gas evolves under its self-consistent electrostatic field and an external magnetic field. This configuration is typical of a tokamak plasma [4,40] where the magnetic field is used to confine particles inside the core of the device. We assume that on the time scale we consider, collisions on ions can be neglected while for electrons, it is entirely modeled with a Fokker–Planck operator. At the formal level, an exhaustive study of asymptotic mass-disparate model with more involved collision operators such as the Boltzmann or Landau operator can be found in the review [16] of Degond. When the mass ratio goes to 0, our model converges to a drift-diffusion equation featuring a magnetic-field dependent diffusion matrix for the electrons coupled with the original kinetic equation for the heavy particles. Similar parabolic equations with non-symmetric diffusion for plasmas can also be found in [5,15,20,17,16].

Hereafter, we start from the physical equations and propose a detailed scaling with respect to the ions time scale. In the dimensionless system, for the light species equation, the leading order terms with respect to the mass ratio are those related to the magnetic field and the collisions. Therefore, the resulting derivation is in the mean time similar to a strong magnetic field limit as in the papers of Golse and Saint-Raymond [29,44], and to a diffusive or parabolic limit as in the work of Poupaud, Soler, Masmoudi and El Ghani in [43,25]. While the latter papers provide us with many tools to handle our own problem, some technical issues in our analysis are closely related to the special features of our model. In a single-species model of charged particles, the other particles density is usually given either as a static regular background or as a function of the electric potential, while our ions are only known to obey a non-trivial kinetic equation. Because of this coupling, it turns out that some extra analysis is needed to recover the regularity required for our limit system to make sense. Besides, we have to control the strong magnetic field. This is done by looking at the interplay between the increasing effect of oscillations and