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Interior regularity for nonlocal fully nonlinear equations with Dini continuous terms

Chenchen Mou

School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332, USA

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Abstract

This paper is concerned with interior regularity of viscosity solutions of non-translation invariant nonlocal fully nonlinear equations with Dini continuous terms. We obtain C^σ regularity estimates for the nonlocal equations by perturbative methods and a version of a recursive Evans–Krylov theorem.

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1. Introduction

In this paper, we investigate interior regularity of viscosity solutions of nonlocal equations of the type

$$\inf_{a\in\mathcal{A}}\left\{\int_{\mathbb{R}^n}\left[u(x+y)-u(x)-\mathbb{1}_{B_1(0)}(y)Du(x)\cdot y\right]K_a(x,y)dy\right\}=f(x),\quad \text{in } B_1(0), \tag{1.1}$$

E-mail address: cmou3@math.gatech.edu.

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where $\mathcal{A}$ is an index set, $\mathbb{1}_{B_1(0)}$ denotes the indicator function of the unit ball $B_1(0)$ and $K_a(x, y)$ is a positive kernel. The kernels $K_a(x, y)$ are symmetric, i.e., for any $x, y \in \mathbb{R}^n$

$$K_a(x, y) = K_a(x, -y), \tag{1.2}$$

and satisfy the uniform ellipticity assumption, i.e., for any $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n \setminus \{0\}$

$$\frac{(2-\sigma)\lambda}{|y|^{n+\sigma}} \le K_a(x, y) \le \frac{(2-\sigma)\Lambda}{|y|^{n+\sigma}}, \tag{1.3}$$

where $0 < \lambda \le \Lambda$. The symmetry assumption is essential for the regularity theory for (1.1), see [25]. Under the symmetry assumption, (1.1) can be rewritten as

$$\inf_{a\in\mathcal{A}} \left\{ \int_{\mathbb{R}^n} \delta u(x, y) K_a(x, y) dy \right\} = f(x), \quad \text{in } B_1(0),$$

where $\delta u(x, y) = u(x + y) + u(x - y) - 2u(x)$. We furthermore assume that the kernels K_a satisfy, for any $x \in \mathbb{R}^n$, any $y \in \mathbb{R}^n \setminus \{0\}$ and $i = 1, 2$

$$|D_y^i K(x, y)| \le \frac{\Lambda(2-\sigma)}{|y|^{n+\sigma+i}}. \tag{1.4}$$

We will obtain C^σ regularity estimates for (1.1) with Dini continuous data in two steps. We first generalize the recursive Evans–Krylov theorem for translation invariant nonlocal fully nonlinear equations from the case of Hölder continuous data, see [15], to the Dini continuous case. We then use the perturbative methods to obtain C^σ regularity estimates for (1.1).

In recent years, regularity theory of viscosity solutions for integro-differential equations has been studied by many authors under uniform ellipticity assumption (1.3). It was initiated by a series of papers [5–7] of L.A. Caffarelli and L. Silvestre, where C^α regularity, $C^{1+\alpha}$ regularity and Evans–Krylov theorem for nonlocal fully nonlinear elliptic equations were established. Later, H. Chang Lara and G. Dávila studied these regularity results for nonlocal fully nonlinear parabolic equations, see in [9–11]. In [18], D. Kriventsov used perturbative methods to prove $C^{1+\alpha}$ regularity estimates for nonlocal fully nonlinear elliptic equations with rough kernels. Then, in [22], J. Serra's results extended the results of [18] to parabolic equations by a Liouville theorem and a blow up and compactness procedure. More recently, in [14], T. Jin and J. Xiong studied $C^{\sigma+\alpha}$ regularity in the x variable for viscosity solutions for linear parabolic integro-differential equations. In [15], T. Jin and J. Xiong proved $C^{\sigma+\alpha}$ regularity estimates for non-translation invariant nonlocal fully nonlinear elliptic equations using a recursive Evans–Krylov theorem and perturbative methods. At the same time, J. Serra refined and improved the method of [22] to obtain $C^{\sigma+\alpha}$ regularity estimates for nonlocal equations with rough kernels, see [23]. The reader can also consult [1,2] for regularity results for a class of second order integro-differential equations with a different uniform ellipticity assumption. It allows nondegeneracy of the nonlocal terms, or nondegeneracy of nonlocal terms in some directions and nondegeneracy of second order terms in the complementary directions. We also refer the reader to [3,4,13,19] for the C^α regularity, $C^{1+\alpha}$ regularity and Evans–Krylov theorem for classical fully nonlinear PDEs.