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Dynamics of a diffusive predator–prey system with a nonlinear growth rate for the predator [☆]

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Abstract

In this paper, we investigate a diffusive predator–prey system with Holling type-II functional response and a nonlinear growth rate for the predator. Our results include the global attractivity of constant equilibria and non-existence of non-constant positive steady states. These results give some ranges for the model parameters within which, spatiotemporal pattern formation is impossible.

MSC: 35K57; 37N25

Keywords: Holling type-II functional response; Steady state; Global attractivity

1. Introduction

One important interaction between biological species is the predator–prey interaction, which can explain, to certain extent, the ecological complexity. Various models described by dynamical systems such as ODE systems and PDE systems have been built to understand predator–prey interaction. For PDE systems, extensive study has produced results regarding boundedness and

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persistence of solutions, global stability of equilibria, and existence and non-existence of non-constant steady state solutions and periodic solutions. See [1–9] and references therein.

A prototypical PDE model is the following diffusive predator–prey system with Holling type-II functional response,

$$\begin{cases} \dfrac{\partial u}{\partial t} - d_1 \Delta u = au\left(1 - \dfrac{u}{k}\right) - \dfrac{buv}{1+mu}, & x \in \Omega,\ t > 0, \\ \dfrac{\partial v}{\partial t} - d_2 \Delta v = -\theta v + \dfrac{cuv}{1+mu}, & x \in \Omega,\ t > 0. \end{cases} \tag{1.1}$$

Here Ω is a bounded domain in $\mathbb{R}^N$ ($N \le 3$) with a smooth boundary $\partial\Omega$; $u(x,t)$ and $v(x,t)$ are the densities of the prey and predator at time $t > 0$ and a spatial position $x \in \Omega$ respectively; $a > 0$ is the intrinsic growth rate of the prey; $k > 0$ is the carrying capacity of the prey; $\theta > 0$ is the mortality rate of the predator; $b > 0$ and $c > 0$ measure the interaction strength between the predator and prey; and $m > 0$ measures the prey's ability to evade attack [10–13]. For the homogeneous Dirichlet boundary condition, Zhou and Mu [14,15] gave the necessary and sufficient conditions for the existence of positive steady states of system (1.1) by fixed point index theory and bifurcation theory. For the homogeneous Neumann boundary condition, by using c as a bifurcation parameter, Yi, Wei and Shi [13] investigated the Hopf and steady state bifurcations of system (1.1) and showed the existence of loops of periodic orbits and steady state solutions. Moreover, Peng and Shi [16] proved the non-existence of non-constant positive steady states when c is sufficiently large, and their result improved the result in [13] and implied that the loops of steady state solutions are bounded. We remark that in [16,13] they actually used an equivalent parameter of c in a nondimensionalized form of system (1.1). In [17], Ko and Ryu investigated the dynamics of system (1.1) with a prey refuge, also under the homogeneous Neumann boundary condition.

For model (1.1), the predator will die out without the prey. Sometimes the predator can also feed on other preys and survive without the specific prey. For example, Du and Lou [10] investigated the following model,

$$\begin{cases} \dfrac{\partial u}{\partial t} - d_1 \Delta u = u(a-u) - \dfrac{buv}{1+mu}, & x \in \Omega,\ t > 0, \\ \dfrac{\partial v}{\partial t} - d_2 \Delta v = v(d-v) + \dfrac{cuv}{1+mu}, & x \in \Omega,\ t > 0. \end{cases} \tag{1.2}$$

If $d > 0$, then the predator can survive without the prey for this model. For the homogeneous Nuemann boundary condition, Du and Lou [10] studied the mutual effect of d and large saturation m on the existence and non-existence of spatially inhomogeneous steady states. Their results depend on diffusion coefficients d_1 and d_2. Recently, Yang, Wu and Nie [18] considered the following diffusive predator–prey system with Holling type-II functional response and a nonlinear growth rate for the predator,

$$\begin{cases} \dfrac{\partial u}{\partial t} - d_1 \Delta u = u(a-ku) - \dfrac{buv}{1+mu}, & x \in \Omega,\ t > 0, \\ \dfrac{\partial v}{\partial t} - d_2 \Delta v = v\left(\dfrac{\alpha}{1+\beta v} - d\right) + \dfrac{cuv}{1+mu}, & x \in \Omega,\ t > 0. \end{cases} \tag{1.3}$$