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A maximal regularity estimate for the non-stationary Stokes equation in the strip

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Abstract

In a d-dimensional strip with $d \geq 2$, we study the non-stationary Stokes equation with no-slip boundary condition in the lower and upper plates and periodic boundary condition in the horizontal directions. In this paper we establish a new maximal regularity estimate in the real interpolation norm

$$\|f\|_{(0,1)} = \inf_{f=f_0+f_1} \left\{ \left\langle \sup_{0<z<1} |f_0| \right\rangle + \left\langle \int_0^1 |f_1| \frac{dz}{(1-z)z} \right\rangle \right\},$$

where the brackets $\langle \cdot \rangle$ denote the horizontal-space and time average. The norms involved in the definition of $\| \cdot \|_{(0,1)}$ are critical for two reasons: the exponents are borderline for the Calderón–Zygmund theory and the weight $1/z$ just fails to be Muckenhoupt. Therefore, the estimate is only true under horizontal bandedness condition (i.e. a restriction to a packet of wave numbers in Fourier space). The motivation to express the maximal regularity in such a norm comes from an application to the Rayleigh–Bénard problem (see [5]).

Keywords: Non-stationary Stokes equations; No-slip boundary condition; Maximal regularity; Real interpolation

Contents

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1. Introduction

In the d-dimensional strip $[0, L)^{d-1} \times [0, 1]$, $d \geq 2$, we consider the non-stationary Stokes equation for the vector field $u(x', z, t)$ and the scalar field $p(x', z, t)$

$$\begin{cases} \partial_t u - \Delta u + \nabla p = f & \text{for} \quad 0 < z < 1, \\ \nabla \cdot u = 0 & \text{for} \quad 0 < z < 1, \\ u = 0 & \text{for} \quad z \in \{0, 1\}, \\ u = 0 & \text{for} \quad t = 0, \end{cases} \tag{1}$$

where $x' \in [0, L)^{d-1}$ and $z \in [0, 1]$ indicate the spatial variables and $t \in \mathbb{R}^+$ denotes the time variable. In what follows it is important to distinguish the horizontal component $u' \in \mathbb{R}^{d-1}$ and the vertical component $u^z \in \mathbb{R}$ of the vector field u.

Motivated by an application to the Rayleigh–Bénard convection problem (see [5]), in this paper we establish the following maximal regularity estimate :

Theorem 1 *(Maximal regularity in the strip). There exists $R_0 \in (0, \infty)$ depending only on d and L such that the following holds. Let u, p, f satisfy the equation (1). Assume f is horizontally band-limited, i.e.*

$$\mathcal{F}' f(k', z, t) = 0 \textit{ unless } 1 \leq R|k'| \leq 4 \textit{ where } R < R_0. \tag{2}$$

Then,

$$\|(\partial_t - \partial_z^2)u'\|_{(0,1)} + \|\nabla'\nabla u'\|_{(0,1)} + \|\partial_t u^z\|_{(0,1)} + \|\nabla^2 u^z\|_{(0,1)} + \|\nabla p\|_{(0,1)} \lesssim \|f\|_{(0,1)}, \tag{3}$$