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Piecewise smooth dynamical systems: Persistence of periodic solutions and normal forms

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Abstract

We consider an n-dimensional piecewise smooth vector field with two zones separated by a hyperplane Σ which admits an invariant hyperplane Ω transversal to Σ containing a period annulus $\mathcal{A}$ fulfilled by crossing periodic solutions. For small discontinuous perturbations of these systems we develop a Melnikov-like function to control the persistence of periodic solutions contained in $\mathcal{A}$. When $n = 3$ we provide normal forms for the piecewise linear case. Finally we apply the Melnikov-like function to study discontinuous perturbations of the given normal forms.

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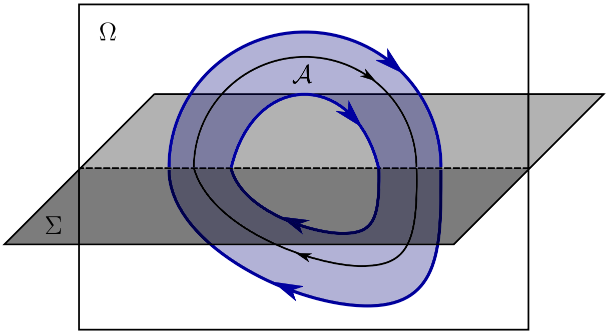

Fig. 1. Representation of an n-dimensional piecewise smooth vector field with two zones separated by the hyperplane Σ having an invariant hyperplane Ω transversal to Σ which possess a period annulus $\mathcal{A}$ fulfilled by crossing periodic solutions.

1. Introduction

The study of the persistence of periodic orbits under small perturbations is a classical and important problem in the qualitative theory of vector fields. For smooth systems this problem was extensively studied at any dimension (see, for instance, the book [9] and the references therein). For nonsmooth systems some techniques to deal with this kind of problem have been recently developed (see, for instance, [18,19]). However, due to the difficulty in applying these last ideas in higher dimensional systems, it has been considered, in general, for planar systems (see, for instance, [2,20,22,23]). As far as we know there are only a few works dealing with this problem in higher dimensional nonsmooth systems (see, for instance, [4]). Therefore, in this paper, our interest lies in studying the persistence of periodic orbits for n-dimensional piecewise smooth vector fields having a period annulus of periodic solutions contained in an invariant hyperplane.

Formally we consider the following (unperturbed) n-dimensional piecewise smooth vector field with two zones separated by the hyperplane $\Sigma = \{z = 0\}$

$$Z_0(\mathbf{x}, z) = \begin{cases} X_0^+(\mathbf{x}, z), & \text{if} \quad z > 0 \\ X_0^-(\mathbf{x}, z), & \text{if} \quad z < 0. \end{cases} \tag{1}$$

Here $(\mathbf{x}, z) \in D \subset \mathbb{R}^{n-1} \times \mathbb{R}$, and $X_0^\pm = (X_{0,1}^\pm, X_{0,2}^\pm, \dots, X_{0,n}^\pm)$. Throughout this paper, X_0^+ and X_0^- will be called, respectively, upper and lower vector fields. As the main hypothesis, we shall assume that there exists an invariant hyperplane $\Omega \subset \mathbb{R}^n$ transversal to Σ containing a period annulus $\mathcal{A}$ fulfilled by crossing periodic solutions of (1) (see Fig. 1).

Particularly for $n = 3$, an interesting case to be considered is when the border of the period annulus $\mathcal{A}$ contains the so called *Teixeira-singularity* (or just *T-singularity*), which represents a source of intricate and complex phenomena [7]. Roughly speaking, the T-singularity is a point where vector field (1) is tangent to both sides of the plane Σ and their orbits nearby return to Σ. In [25], for a particular definition of structural stability of nonsmooth systems, Teixeira showed that the T-singularity is not structurally stable, and asymptotic stability is determined only under limited conditions of hyperbolicity. Recently, several other aspects of the dynamic of nonsmooth systems in the presence of a T-singularity has been studied, see, for instance, [6,15,16] and the references therein.