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Nonplanar traveling fronts in reaction–diffusion equations with combustion and degenerate Fisher-KPP nonlinearities

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Abstract

This paper is concerned with nonplanar traveling fronts in reaction–diffusion equations with combustion nonlinearity and degenerate Fisher-KPP nonlinearity. Our study contains two parts: in the first part we establish the existence of traveling fronts of pyramidal shape in $\mathbb{R}^3$, and in the second part we establish the existence and stability of V-shaped traveling fronts in $\mathbb{R}^2$.

MSC: 35K57; 35C07; 35B35; 35B40

Keywords: Reaction–diffusion equations; Traveling fronts of pyramidal shapes; V-shaped traveling fronts; Combustion nonlinearity; Degenerate Fisher-KPP nonlinearity

1. Introduction

In this paper we investigate nonplanar traveling fronts of the following equation

$$\frac{\partial}{\partial t}u(\mathbf{x},t) = \Delta u(\mathbf{x},t) + f(u), \quad \mathbf{x} \in \mathbb{R}^N,\ t > 0, \tag{1.1}$$

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where $N \in \mathbb{N}$. We mainly consider two types of nonlinear reaction terms f, namely, combustion nonlinearity and monostable nonlinearity. In order to motivate our study, we first show some known results and open questions in the study of nonplanar traveling fronts of (1.1) with such two classes of reaction terms, respectively.

- *Combustion nonlinearity*. In this case the function $f(u)$ satisfies

$$\exists \theta \in (0,1),\ f(u) = 0 \text{ for all } u \in [0,\theta] \cup \{1\},\ f(u) > 0 \text{ for all } u \in (\theta,1),\ f'(1) < 0.$$

For such type of nonlinearity, Bonnet and Hamel [1] established the existence of two-dimensional V-shaped traveling fronts of (1.1) with $N = 2$, which is the first rigorous analysis of the conical premixed Bunsen flames. After that, Hamel and Monneau [14] investigated conical traveling fronts of (1.1) with $N \geq 2$. However, not only the conical flames, but also flames of polyhedral shapes or flames of various kinds of smooth shapes have been found by experimental observations and numerical calculations for the Bunsen burners, see Buckmaster [3], Gutman et al. [12], Smith and Pickering [35], and Olagunju and Matkowsky [31]. It is clear that giving a rigorous analysis for flames of polyhedral shapes by equation (1.1) is equivalent to establishing the existence of traveling fronts of convex polyhedral shapes for equation (1.1) (see also Taniguchi [38, Section 8]). This leads to the first question:

Question 1. *How to establish the existence of traveling fronts of convex polyhedral shapes for equation (1.1) with combustion nonlinearity?*

Here we would like to point out that traveling fronts of convex polyhedral shapes have been established for equation (1.1) with bistable nonlinearity, see Taniguchi [36–39]. For more results on nonplanar traveling fronts of equation (1.1) with bistable nonlinearity, we refer to [7,8,11,13,16,17,19–21,23,27,29,30,34]. For nonplanar traveling fronts of bistable reaction–diffusion systems and periodic reaction–diffusion equations, we refer to [28,33,40–44].

On the other hand, the stability of nonplanar traveling fronts is also an important subject. In two-dimensional space, Hamel et al. [15] proved the asymptotic stability of V-shaped traveling fronts of (1.1) with combustion nonlinearity under the condition that the initial value is less than the V-shaped traveling fronts (see Hamel et al. [15, Theorem 1.6]). Thus we have the following question:

Question 2. *Are the V-shaped traveling fronts stable for equation (1.1) with combustion nonlinearity if the initial values are greater than the V-shaped traveling fronts?*

- *Monostable nonlinearity*. In this case the function $f(u)$ satisfies

$$f(0) = f(1) = 0,\ f(u) > 0 \text{ for all } u \in (0,1),\ f'(1) < 0.$$

A typically monostable nonlinearity is of Fisher-KPP type, namely, the function f further satisfies $f'(0) > 0$ and $f''(u) \leq 0$ in $u \in (0,1)$. An example of Fisher-KPP nonlinearities is the function $f(u) = u(1-u)$. For such nonlinearity, Hamel and Nadirashvili [18] established the existence of nonplanar traveling fronts of (1.1). Furthermore, Huang [22] established the stability of the nonplanar traveling fronts. For more results on curved fronts of Fisher-KPP equations,