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Analysis of a coupled spin drift–diffusion Maxwell–Landau–Lifshitz system ☆

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Abstract

The existence of global weak solutions to a coupled spin drift–diffusion and Maxwell–Landau–Lifshitz system is proved. The equations are considered in a two-dimensional magnetic layer structure and are supplemented with Dirichlet–Neumann boundary conditions. The spin drift–diffusion model for the charge density and spin density vector is the diffusion limit of a spinorial Boltzmann equation for a vanishing spin polarization constant. The Maxwell–Landau–Lifshitz system consists of the time-dependent Maxwell equations for the electric and magnetic fields and of the Landau–Lifshitz–Gilbert equation for the local magnetization, involving the interaction between magnetization and spin density vector. The existence proof is based on a regularization procedure, L^2-type estimates, and Moser-type iterations which yield the boundedness of the charge and spin densities. Furthermore, the free energy is shown to be nonincreasing in time if the magnetization–spin interaction constant in the Landau–Lifshitz equation is sufficiently small.

MSC: 35K51; 35Q61; 35Q60; 82D40

Keywords: Spin drift–diffusion equations; Maxwell–Landau–Lifshitz system; Existence of weak solutions; Von-Neumann entropy; Bounded weak solutions

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1. Introduction

Magnetic devices, such as magnetic sensors and hard disk read heads, typically consist of ferromagnetic/nonmagnetic layer structures. A model for magnetic multi-layers was first introduced by Slonczewski [33]. This model is well suited for Magnetoresistive Random Access Memory (MRAM) devices but it is less appropriate for current-driven domain wall-motion. A more general approach is to introduce the spin accumulation coupled to the magnetization dynamics. The evolution of the magnetization is modeled by the Landau–Lifshitz (–Gilbert) equation [36]. When electrodynamic effects cannot be neglected (like in high-frequency regimes), this description needs to be coupled to the Maxwell equations. In this paper, we analyze for the first time a coupled spin drift–diffusion Maxwell–Landau–Lifshitz system in two space dimensions with physically motivated boundary conditions.

Let us describe our model in more detail. We consider a three-layer semiconductor structure $\Omega \subset \mathbb{R}^2$ consisting of two ferromagnetic regions $\omega_1, \omega_2 \subset \Omega$, separated by a nonmagnetic interlayer $\Omega \backslash \omega$, where $\omega = \omega_1 \cup \omega_2$ is the union of magnetic layers [1].

Landau–Lifshitz–Gilbert equation. The dynamics of the magnetization $\mathbf{m} = (m_1, m_2, m_3)$ is governed by the Landau–Lifshitz–Gilbert (LLG) equation

$$\partial_t \mathbf{m} = \mathbf{m} \times (\Delta \mathbf{m} + \mathbf{H} + \beta \mathbf{s}) - \alpha \mathbf{m} \times (\mathbf{m} \times (\Delta \mathbf{m} + \mathbf{H} + \beta \mathbf{s})) \quad \text{in } \omega, \ t > 0, \tag{1}$$

where the effective field $\mathbf{H}_{\text{eff}} = \Delta \mathbf{m} + \mathbf{H}$ consists of the sum of the exchange field contribution $\Delta \mathbf{m}$ and the magnetic field $\mathbf{H}$, and $\alpha > 0$ denotes the Gilbert damping constant. The additional term $\beta \mathbf{s}$ models the interaction between the magnetization $\mathbf{m}$ and spin accumulation $\mathbf{s}$ with strength $\beta > 0$ [9,36]. We choose the initial and boundary conditions

$$\mathbf{m}(0) = \mathbf{m}^0 \quad \text{in } \omega, \qquad \nabla \mathbf{m} \cdot \boldsymbol{\nu} = 0 \quad \text{on } \omega, \ t > 0, \tag{2}$$

where $\boldsymbol{\nu}$ is the outward unit normal on $\partial\omega$, we write $\mathbf{m}(0) = \mathbf{m}(\cdot, 0)$, and the notation $\nabla \mathbf{m} \cdot \boldsymbol{\nu} = 0$ means that $\nabla m_i \cdot \boldsymbol{\nu} = 0$ for $i = 1, 2, 3$. The Neumann conditions were also used in, e.g., [1,17]. We set $\mathbf{m} = 0$ in $\Omega \backslash \omega$.

The existence and non-uniqueness of weak solutions to the LLG equation goes back to [3,34]. The local existence of a unique strong solution was proven in [5]. In two space dimensions and for sufficiently small initial data, the strong solution is, in fact, global in time [5]. For general initial data, the two-dimensional solution may develop finitely many point singularities after finite time; see [20] for a discussion. The existence of weak solutions in three space dimensions with physically motivated boundary conditions was shown in [4], based on a finite-element approximation. For a complete review on analytical results, we refer to [10,26].

Maxwell equations. The Maxwell equations are given by the time-dependent Ampère and Faraday laws for the electric and magnetic fields $\mathbf{E} = (E_1, E_2, E_3)$ and $\mathbf{H} = (H_1, H_2, H_3)$, respectively,

$$\partial_t \mathbf{E} - \operatorname{curl} \mathbf{H} = \mathbf{J}_e, \qquad \partial_t \mathbf{H} + \operatorname{curl} \mathbf{E} = -\partial_t \mathbf{m} \quad \text{in } \Omega, \ t > 0, \tag{3}$$

and by the Gauss laws

$$\operatorname{div} \mathbf{E} = \rho - C(x), \qquad \operatorname{div}(\mathbf{H} + \mathbf{m}) = 0 \quad \text{in } \Omega, \ t > 0. \tag{4}$$