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Ground state solutions for some Schrödinger–Poisson systems with periodic potentials ☆

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Abstract

In this paper, we consider the following nonlinear Schrödinger–Poisson system

$$\begin{cases} -\Delta u + V(x)u + \phi u = f(x,u), & \text{in } \mathbb{R}^3, \\ -\Delta \phi = u^2, & \text{in } \mathbb{R}^3, \end{cases}$$

where the nonlinearity f is superlinear at infinity with subcritical or critical growth and V is positive, continuous and periodic in x. The existence of ground state solutions, i.e., nontrivial solutions with least possible energy of this system is obtained. Moreover, when $V \equiv 1$, we obtain ground state solutions for the above system with a wide class of superlinear nonlinearities by using a new approach.

Keywords: Schrödinger–Poisson systems; Ground state; Nehari manifold; Critical exponent

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1. Introduction and main results

In this paper, we are concerned with the following Schrödinger–Poisson system

$$\begin{cases} -\Delta u + V(x)u + \phi u = f(x,u), & \text{in } \mathbb{R}^3, \\ -\Delta \phi = u^2, & \text{in } \mathbb{R}^3, \end{cases} \tag{1.1}$$

where V satisfies

(V) V is continuous, 1-periodic in x_i for $i = 1, 2, 3$ and $V_0 = \inf_{x\in\mathbb{R}^3} V(x) > 0$.

Systems like (1.1) have been widely investigated because they have a strong physical meaning. It is well known that they arise in quantum mechanics model [7] describing solitary waves for nonlinear stationary Schrödinger type equations interacting with an electrostatic field. We refer to [7,8] and the references cited therein for more physical background.

In recent years, there have been enormous results on existence, nonexistence and multiplicity of solutions for such system depending on the assumptions of the potential V. The greatest part of the literature focuses on the study of problem (1.1) with $V \equiv 1$, see e.g. [5,23] for the pure power type nonlinearity $|u|^{p-2}u$ with $p \in (2, 6)$, [11] for the case of $f(x,u) = a(x)|u|^{p-2}u$, $p \in (4, 6)$ with suitable assumptions on nonnegative function a without any symmetry property, [16] for similar problem with $f(x,u) = a(x)|u|^{p-2}u + \mu h(x)u$ where $p \in (4, 6)$, a changes sign satisfying $a(x) \to a_\infty < 0$ as $|x| \to \infty$ and $\mu > \mu_1$ but near μ_1 where μ_1 is the first eigenvalue of $-\Delta + id$ in $H^1(\mathbb{R}^3)$ with positive weight function h, [32] for system (1.1) with $f(x,u) = K(x)|u|^4u + \mu Q(x)|u|^{q-2}u$ where $q \in [4, 6)$ and $\mu > 0$. The proofs in [5,11,23,32] are based on minimizing the associated functional restricted to a suitable manifold $\mathcal{M}$ which is C^1. When V is a radially symmetric function, Ambrosetti and Ruiz [4] obtained multiple bound states for system (1.1) with the pure power nonlinearity $|u|^{p-2}u$, $2 < p < 6$ by working in $H^1_r(\mathbb{R}^3)$, the subspace of radial functions of $H^1(\mathbb{R}^3)$. In [5], Azzollini and Pomponio proved the existence of a ground state solution of (1.1) for the case that $f(x,u) = |u|^{p-1}u$ with $p \in (3, 5)$ if V satisfies

(V_1) $V_\infty = \lim_{|y|\to\infty} V(y) \geq V(x)$ a.e. in $x \in \mathbb{R}^3$, and the strict inequality holds on a set of positive measure.

It is well known that (V_1) plays a crucial role in using Lions' concentration-compactness principle. It was also proved in [31] that system (1.1) still admits a ground state solution for $p \in (2, 3]$ if (V_1) holds and V is weakly differentiable satisfying some additional conditions. More recently, replacing V by λV, under suitable assumptions on V, the steep potential well case was considered for system (1.1) with $f(x,u) = |u|^{p-2}u$ for p varying in the interval $(2, 6)$ in [20,33] and the existence and concentration results were obtained. Moreover, many researches have devoted to the study of a certain concentration phenomena for the so-called semi-classical limit of system (1.1), that is the first equation looks like $-\varepsilon^2\Delta u + V(x)u + K(x)\phi u = f(x,u)$, see e.g. [3,13,15,17,22,24,28] and the references therein.

Our argument is variational. Let $E := H^1(\mathbb{R}^3)$. Under (V), we can define a new norm

$$\|u\| := \langle u, u\rangle^{1/2} = \left(\int_{\mathbb{R}^3} |\nabla u|^2 + V(x)u^2\right)^{1/2}$$