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Asymptotic dynamics on a singular chemotaxis system modeling onset of tumor angiogenesis

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Abstract

The asymptotic behavior of solutions to a singular chemotaxis system modeling the onset of tumor angiogenesis in two and three dimensional whole spaces is investigated in the paper. By a Cole–Hopf type transformation, the singular chemotaxis is converted into a non-singular hyperbolic system. Then we study the transformed system and establish the global existence, asymptotic decay rates and diffusion convergence rate of solutions by the method of energy estimates. The main novelty of our results is the finding of a hidden interactive dissipation structure in the system by which the energy dissipation is established.

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Keywords: Chemotaxis; Tumor angiogenesis; Energy estimates; Global well-posedness; Decay estimates

1. Introduction

It is widely recognized that tumor angiogenesis plays a central role in spreading cancer cells to other tissues in cancer metastasis, and hence making cancer a potentially life-threatening disease. Therefore it is of great importance and interest to understand the underlying mechanism of

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tumor angiogenesis which starts with cancerous tumor cells releasing signaling molecules vascular endothelial growth factor (VEGF) to surrounding normal host tissue and activate the motion of vascular endothelial cells. To capture the main interaction between VEGF and vascular endothelial cells, the following PDE model was proposed in [12]

$$\begin{cases} u_t = \nabla \cdot \big(D\nabla u - \chi u \nabla \ln c\big), \\ c_t = \varepsilon \Delta c - \mu u c \end{cases} \tag{1.1}$$

where $u(x,t)$ and $c(x,t)$ denote the density of vascular endothelial cells and concentration of VEGF, respectively. The parameter $D > 0$ is the diffusivity of endothelial cells, $\chi > 0$ is referred to as the chemotactic coefficient measuring the intensity of chemotaxis and μ denotes the degradation rate of the chemical (VEGF) c. The parameter $\varepsilon \geq 0$ denotes the chemical diffusion rate and could be small or negligible since the chemical diffusion is far less important than its interaction with endothelial cells as treated in [12]. For more information on the cancer modeling, we refer to a review paper [4] and the references therein. Except the afore-mentioned applications, the model (1.1) was also previously considered in [22] to examine the boundary movement of bacterial population chemotaxis, and a specialized case investigated in [21,27] for traveling wave solutions.

The striking feature of model (1.1) is that the first equation contains a logarithmic sensitivity function $\ln c$ which is singular at $c = 0$. This singular logarithmic sensitivity was first used by Keller and Segel in their seminal paper [10] to describe the propagation of traveling wave band formed by bacterial chemotaxis observed in the experiment of Adler [1]. Its mathematical derivation was later given in [23] and biological basis was provided in [9] by both experimental measurements and model simulations. Therefore the logarithmic sensitivity is meaningful both mathematically and biologically though it causes great difficulties in its mathematical analysis and numerical computations. Among other things, the foremost mathematical question is therefore how to resolve the singularity. Toward this end, a Cole–Hopf type transformation as follows was used in [11,31]

$$\mathbf{v} = -\nabla \ln c = -\frac{\nabla c}{c} \tag{1.2}$$

which, together with scalings $\tilde{t} = \frac{\chi\mu}{D} t$, $\tilde{x} = \frac{\sqrt{\chi\mu}}{D} x$, $\tilde{\mathbf{v}} = \sqrt{\frac{\chi}{\mu}} \mathbf{v}$, transforms the system (1.1) into a hyperbolic system:

$$\begin{cases} u_t - \Delta u = \nabla \cdot (u\mathbf{v}), & x \in \Omega,\ t > 0, \\ \mathbf{v}_t - \varepsilon \Delta \mathbf{v} = \nabla(-\varepsilon|\mathbf{v}|^2 + u), & x \in \Omega,\ t > 0, \\ (u, \mathbf{v})(x, 0) = (u_0, \mathbf{v}_0)(x), & x \in \Omega, \end{cases} \tag{1.3}$$

where tildes have been dropped for convenience and Ω is either the whole space or a bounded domain with smooth boundary. Compared to the original model (1.1), the transformed system (1.3) is much more manipulable mathematically since the singularity vanishes. There was an amount of interesting works carried out for the transformed system (1.3) and hence for the original model (1.1) by reverting the Cole–Hopf transformation (1.2). We briefly review these results below by the nature of domain. First in the one dimensional bounded domain $\Omega \subset \mathbb{R}$, the global existence of solutions of (1.3) with $\varepsilon = 0$ subject to Neumann–Dirichlet boundary condition was first established in [32] for small data, and later in [29] for large data with any $\varepsilon \geq 0$. Recently the