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# Higher-order resonances and instability of high-frequency WKB solutions

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**Abstract**

This paper focuses on the destabilizing role of resonances in high-frequency WKB solutions. Specifically, we study higher-order resonances associated with higher-order harmonics generated by nonlinearities. We give examples of systems and solutions for which such resonances generate instantaneous instabilities, even though the equations linearized around the leading WKB terms are initially stable, meaning in particular that the key destabilizing terms are not present in the data.


*MSC:* 35B40; 35Q60

*Keywords:* Higher-order resonances; WKB solutions; Transparency conditions; Instabilities

**Contents**



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## 1. Introduction

We study highly-oscillating solutions to hyperbolic systems based on Maxwell's equations. Considerable progress has recently been made in this line of research, especially following the works of Joly, Métivier and Rauch in the nineties (see for instance [3,11,12], and [4] for an overview and further references). The underlying physical problems deal with light–matter interactions.