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Degenerate two-phase porous media flow model with dynamic capillarity

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Abstract

In this paper, we investigate a degenerate elliptic–parabolic system which describes the flow of two incompressible, immiscible fluids in porous media and including dynamic effects in the phase-pressure difference. First, for the regularized case, the existence and uniqueness of the weak solution are obtained. Then we let the regularization parameter go to zero to show the existence of weak solutions under degenerate case.

Keywords: Degeneracy; Dynamic capillary pressure; Two-phase flow; Weak solution; Uniqueness; Pseudo-parabolic system

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1. Introduction

We analyze the existence and, where appropriate, uniqueness of a weak solution to the elliptic–parabolic system

$$\partial_t u + \nabla \cdot (k_n(u)\nabla p) - \Delta\theta(u) = 0, \tag{1.1}$$

$$\nabla \cdot (k(u)\nabla p) + \nabla \cdot (k_w(u)\nabla(\tau(u)\partial_t u)) = 0, \tag{1.2}$$

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with $k = k_w + k_n$. The equations hold in $Q := (0, T_M] \times \Omega$. Here Ω is a bounded domain in $\mathbb{R}^d$ ($d = 1, 2, 3$), having Lipschitz continuous boundary, and $T_M > 0$ is a given maximal time. The unknowns are u and p. The work is motivated by two-phase flow in porous media (e.g. oil and water).

1.1. Two phase flow model in porous media under non-equilibrium condition

The system (1.1)–(1.2) models two phase flow in porous media with dynamic effects in the phase pressure difference. It is obtained by including Darcy's law for both phases in the mass conservation laws. With w, n being indices for the wetting, respectively, non-wetting phase, the mass conservation equations are (see [4,24,31])

$$\phi \frac{\partial s_\alpha}{\partial t} + \nabla \cdot q_\alpha = 0, \quad \alpha = w, n. \tag{1.3}$$

The coefficient ϕ represents the porosity of the porous medium, while s_α and q_α denote the saturation and the volumetric velocity of the α phase. The volumetric velocity q_α is deduced from the Darcy's law as

$$q_\alpha = -\frac{\bar{k}}{\mu_\alpha} k_{r\alpha}(s_\alpha) \nabla p_\alpha, \quad \alpha = w, n, \tag{1.4}$$

where $\bar{k}$ is the absolute permeability of the porous medium, p_α the pressure, μ_α the viscosity and $k_{r\alpha}$ the relative permeability of the α phase. The specific function of $k_{r\alpha}$ is assumed to be known. Substituting (1.4) in (1.3) gives

$$\phi \frac{\partial s_\alpha}{\partial t} - \nabla \cdot \left(\frac{\bar{k} k_{r\alpha}}{\mu_\alpha} \nabla p_\alpha\right) = 0, \quad \alpha = w, n. \tag{1.5}$$

We assume that only two phases are present

$$s_w + s_n = 1. \tag{1.6}$$

To complete the model, one commonly assumes a relationship between the phase pressure difference and s_w. Under equilibrium assumption, this is

$$p_n - p_w = p_c(s_w),$$

with a given function $p_c = p_c(\cdot)$. Experimental results [17] have, however, proved the limitation of this assumption. Alternatively, in [23] the following relation is proposed

$$p_n - p_w = p_c(s_w) - \tau(s_w) \frac{\partial s_w}{\partial t}. \tag{1.7}$$

The damping function τ as well as the function p_c, which represents the capillary pressure under equilibrium condition, are assumed to be known. Summing the two equations from (1.5) and making use of (1.6) gives