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J. Differential Equations 259 (2015) 454–472

Journal of Differential Equations

www.elsevier.com/locate/jde

Stabilization of semilinear heat equations, with fading memory, by boundary feedbacks

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Received 2 December 2013; revised 9 February 2015

Available online 24 February 2015

Abstract

In this paper, we design stabilizing Dirichlet boundary feedback laws for heat equations with fading memory. The linear finite-dimensional controllers are easily manageable from the computational point of view, because of their simple feedback form, involving only the first $N \in \mathbb{N}$ eigenfunctions of the Laplace operator. Two examples are provided at the end of the paper, in order to illustrate the acquired results.

MSC: 45K05; 80A20; 93D15

Keywords: Heat equation with memory; Stabilization; Feedback controller; Laplace operator; Spectral decomposition

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http://dx.doi.org/10.1016/j.jde.2015.02.010

1. Introduction

The aim of the present paper is to study the Dirichlet boundary control problem of the following integro-partial differential evolution equation:

$$\begin{cases} \partial_t y(t,x) = \Delta y(t,x) + \int\limits_{-\infty}^{t} k(t-s)\Delta y(s,x) + \mu \int\limits_{-\infty}^{t} k(t-s) y(s,x) ds \\ \qquad\qquad + f(y(t,x)), \ (t,x) \in Q := (0,\infty)\times\Omega, \\ y(t,x) = u(t,x) \text{ on } \Sigma_1 := (0,\infty)\times\Gamma_1, \\ \frac{\partial y}{\partial \mathbf{n}} = 0 \text{ on } \Sigma_2 := (0,\infty)\times\Gamma_2, \\ y(t,x) = y_o(t,x), (t,x) \in (-\infty, 0]\times\Omega. \end{cases} \tag{1.1}$$

Here, Ω is a bounded and open domain of $\mathbb{R}^d$, $d = 1,2,3$, with smooth boundary $\partial\Omega = \Gamma_1 \cup \Gamma_2$; $\mathbf{n}$ is the outward unit normal on the boundary $\partial\Omega$. The unknown scalar function $y = y(t,x)$ depends on the time $t \in \mathbb{R}$ and the spatial variable $x \in \Omega$. In Eq. (1.1), Δ denotes the Laplace operator in $\mathbb{R}^d$. The effects of memory are expressed in the linear time convolution of the functions $\Delta y(\cdot,\cdot)$, respectively $y(\cdot,\cdot)$, and the memory kernel $k(\cdot)$. The Dirichlet controller u is applied on Γ_1, while Γ_2 is insulated.

As it can be seen in the initial condition $(1.1)_4$, it is assumed that the function $y(t,x)$ is known for all $t \leq 0$. However, $y(t,x)$ does not necessarily satisfy the equation for negative t. On the nonlinear function f we shall assume that $f(0) = 0$ and $f'(0) > 0$, and choose from the following two hypotheses

(f_1) $f \in C^1(\mathbb{R})$;
(f_2) $f \in C^2(\mathbb{R})$, and there exist $C_1 > 0$, $q \in \mathbb{N}$, $\alpha_i > 0$, $i = 1,\dots,q$, when $d = 1,2$, and $0 < \alpha_i \leq 1$, $i = 1,\dots,q$, when $d = 3$, such that

$$|f''(y)| \leq C_1 \left(\sum_{i=1}^{q} |y|^{\alpha_i} + 1 \right), \ \forall y \in \mathbb{R}.$$

(Here f' stands for the derivative $\frac{d}{dy} f$. In some occasions, we shall still denote by $'$ the derivative with respect to time, i.e., $\frac{d}{dt}$, but this will be clarified from the context.) Finally, μ is some nonnegative constant.

The aim of this paper is to study the stabilizability of the null solution in the nonlinear system (1.1). The instability may occur because of the presence of the nonlinear term f. Besides this, since we assume that the memory kernel k is positive, the instability may be also caused by the presence of the memory term containing the positive constant μ.

This model was introduced in [20]. Eq. (1.1) describes the heat flow in a rigid, isotropic, homogeneous heat conductor with memory. It is derived in the framework of the theory of heat flows with memory established in [12]. Similar equations were considered in different papers, but, the problem of behavior of solutions and stability was directly addressed in [11,17,18]. There, the main ingredient used by the authors is the so-called history space setting which consists in considering some past history variables as additional components of the phase space corresponding to the equation under study (this idea is due to Dafermos [13]). In the present paper, we do not appeal to it, but, to the theory of nonlinear Volterra equations with positive kernels (see [8]). We shall study the null boundary stabilization problem of (1.1). More precisely, we shall construct