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Multifunctions of bounded variation

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Abstract

Consider control systems described by a differential equation with a control term or, more generally, by a differential inclusion with velocity set $F(t, x)$. Certain properties of state trajectories can be derived when it is assumed that $F(t, x)$ is merely measurable w.r.t. the time variable t. But sometimes a refined analysis requires the imposition of stronger hypotheses regarding the time dependence. Stronger forms of necessary conditions for minimizing state trajectories can be derived, for example, when $F(t, x)$ is Lipschitz continuous w.r.t. time. It has recently become apparent that significant addition properties of state trajectories can still be derived, when the Lipschitz continuity hypothesis is replaced by the weaker requirement that $F(t, x)$ has bounded variation w.r.t. time. This paper introduces a new concept of multifunctions $F(t, x)$ that have bounded variation w.r.t. time near a given state trajectory, of special relevance to control. We provide an application to sensitivity analysis.
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1. Introduction

A widely used framework for control systems analysis is based on a description of the dynamic constraint in the form of a differential inclusion

$$\dot{x}(t) \in F(t, x(t)) \quad \text{a.e. } t \in [S, T], \tag{1.1}$$

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in which $F(\cdot,\cdot) : [S,T] \times \mathbb{R}^n \rightsquigarrow \mathbb{R}^n$ is a given multifunction. We refer to absolutely continuous functions $x(\cdot) : [S,T] \to \mathbb{R}^n$ satisfying (1.1) as state trajectories.

It is well known that the assumptions that are made regarding the t dependence of $F(t,x)$ have a critical effect on the qualitative properties of the set of state trajectories and, if state trajectories minimizing a given cost function are of primary interest, the assumptions affect the regularity properties of the value function, the nature of necessary conditions that can be derived, etc. In previous research on the distinct properties of state trajectories, depending on the different assumptions that are made about the regularity of $F(t,x)$ with respect to t, attention has focused on consequences of hypothesizing:

(a): $t \to F(t,x)$ is measurable, or
(b): $t \to F(t,x)$ is Lipschitz continuous.

(See [12, Chap. 2] for definitions of measurability and Lipschitz continuity of multifunctions.) Some examples of distinct properties are as follows.

- (i): Standard necessary conditions of optimality, in state-constrained optimal control, take a non-degenerate form, under the assumption that $F(\cdot,x)$ is Lipschitz continuous and other assumptions, but this is no longer in general the case if $F(\cdot,x)$ is merely measurable [12, Thm. 10.6.1].
- (ii): Optimal state trajectories have essentially bounded derivatives under the assumption that $F(\cdot,x)$ is Lipschitz continuous and other assumptions, but may not be essentially bounded if $F(\cdot,x)$ is merely measurable [7].
- (iii): The Hamiltonian evaluated along an optimal state trajectory and co-state trajectory cannot contain jumps if $F(\cdot,x)$ is Lipschitz continuous, but may be discontinuous if $F(\cdot,x)$ is merely measurable [5].

Other examples where there are significant differences in the implications of the two kinds of regularity hypotheses arise in the study of regularity properties of the value function for state constrained optimal control problems [3], validity of necessary conditions of optimality for free-time optimal control problems [5], the interpretation of costate trajectories as gradients of the value function [2] and in more general sensitivity analysis.

Are there other classes of differential inclusions $F(t,x)$, defined by their regularity w.r.t. t, where interesting, distinct properties are encountered? It turns out that multifunctions $F(t,x)$ having bounded variation w.r.t. t is an example of such a class. Many properties of the set of state trajectories that are valid when $F(t,x)$ has Lipschitz t-dependence, but not in general when $F(t,x)$ has measurable t-dependence, have analogues when $F(t,x)$ has bounded variation t-dependence.

How should we define '$F(t,x)$ has bounded variation t-dependence'? An obvious approach is to require:

$$\sup\left\{\sum_{i=0}^{N-1} \sup_{x\in X} d_H(F(t_{i+1},x), F(t_i,x))\right\} < \infty\,. \tag{1.2}$$

Here, X is some suitably large subset of $\mathbb{R}^n$. $d_H(\cdot,\cdot)$ denotes the Hausdorff distance. (See (1.4)). The outer supremum is taken over all possible partitions $\{t_0 = S, \ldots, t_N = T\}$ of $[S,T]$. But we follow a more refined approach, for reasons that we now describe.

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