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Strong solutions to the density-dependent incompressible nematic liquid crystal flows

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Abstract

In this paper, we investigate the density-dependent incompressible nematic liquid crystal flows in n-dimensional ($n = 2$ or 3) bounded domain. The local existence and uniqueness of strong solutions are obtained when the viscosity coefficient of fluid depends on density. Furthermore, one establishes blowup criterions for the regularity of the strong solutions in dimensions two and three respectively. In particular, we build a blowup criterion just in terms of the gradient of density if the initial direction field satisfies some geometric configuration. For these results, the initial density need not be strictly positive.

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Keywords: Incompressible nematic liquid crystal flows; Density-dependent; Strong solutions; Blowup criterion

1. Introduction

Nematic liquid crystals contain a large number of elongated, rod-like molecules and possess the same orientational order. The continuum theory of liquid crystals due to Ericksen [1] and Leslie [2] was developed around 1960s, see also [3]. Since then, numerous researchers have obtained some important developments for liquid crystals not only in theory but also in the

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application. When the fluid containing nematic liquid crystal materials is at rest, we have the well-known Ossen–Frank theory for static nematic liquid crystals, see the pioneering work by Hardt et al. [4] on the analysis of energy minimal configurations of nematic liquid crystals. Generally speaking, the motion of fluid always takes place. The so-called Ericksen–Leslie system is a macroscopic description of the time evolution of the materials under the influence of both the flow velocity field and the macroscopic description of the microscopic orientation configuration of rod-like liquid crystals. In this paper, we investigate the motion of incompressible nematic liquid crystal flows, which are described by the following simplified version of the Ericksen–Leslie equations:

$$\begin{cases} \rho_t + \operatorname{div}(\rho u) = 0, \\ (\rho u)_t + \operatorname{div}(\rho u \otimes u) - \operatorname{div}(2\mu(\rho) D(u)) + \nabla P = -\lambda \operatorname{div}(\nabla d \odot \nabla d), \\ \operatorname{div} u = 0, \\ d_t + u \cdot \nabla d = \theta(\Delta d + |\nabla d|^2 d), \end{cases} \tag{1.1}$$

in $\Omega \times (0, +\infty)$, where Ω is a bounded domain with smooth boundary in $\mathbb{R}^n$ ($n = 2$ or 3). Here ρ, u, P and d denote the unknown density, velocity, pressure and macroscopic average of the nematic liquid crystal orientation respectively. $D(u) = \frac{\nabla u + \nabla^T u}{2}$ is the deformation tensor, where ∇u presents the gradient matrix of u and $\nabla^T u$ is its transpose. $\mu > 0$, $\lambda > 0$, $\theta > 0$ are viscosity of fluid, competition between kinetic and potential energy, and microscopic elastic relaxation time respectively. The viscosity coefficient $\mu = \mu(\rho)$ is a general function of density, which is assumed to satisfy

$$\mu \in C^1[0, \infty) \quad \text{and} \quad \mu \geq \underline{\mu} > 0 \quad \text{on} \quad [0, \infty), \tag{1.2}$$

for some positive constant $\underline{\mu}$. Without loss of generality, both λ and θ are normalized to 1. The symbol $\nabla d \odot \nabla d$ denotes the $n \times n$ matrix whose (i, j)-th entry is given by $\nabla d_i \cdot \nabla d_j$, for $i, j = 1, 2, \ldots, n$. To complete the equations (1.1), we consider an initial boundary value problem for (1.1) with the following initial and boundary conditions

$$(\rho, u, d)|_{t=0} = (\rho_0, u_0, d_0), \ |d_0| = 1, \ \text{in } \Omega; \tag{1.3}$$

$$u = 0, \quad \frac{\partial d}{\partial \nu} = 0 \quad \text{on } \partial\Omega; \tag{1.4}$$

where ν is the unit outward normal vector to $\partial\Omega$.

When the fluid is the homogeneous case, the systems (1.1)–(1.4) are the simplified model of nematic liquid crystals with constant density. If the term $|\nabla d|^2 d$ be replaced by the Ginzburg–Laudan type approximation term $\frac{1-|d|^2}{\varepsilon^2} d$, Lin [5] first derived a simplified Ericksen–Leslie equations modeling the liquid crystal flows in 1989. Later, Lin and Liu [6,7] made some important analytic studies, such as the existence of weak/strong solutions and the partial regularity of suitable solutions. Recently, Dai et al. [8] studied the large time behavior of solutions and gave the decay rate with small initial data in the three dimensional whole space $\mathbb{R}^3$. On the other hand, Grasselli and Wu [9] considered the long time behavior of solutions and obtained the estimates on the convergence rates with external force. They also showed the existence of global strong solutions provided that either the viscosity is large enough or the initial datum is closed to a