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On the behavior of boundary layers of one-dimensional isentropic planar MHD equations with vanishing shear viscosity limit ☆

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Abstract

In this paper, we consider an initial-boundary value problem for the one-dimensional equations of planar isentropic magnetohydrodynamics (MHD). The vanishing shear viscosity limit is justified. More important, both the thickness and the behavior of boundary layers are described. The proofs of these results are based on the full use of the material derivative, the "effective viscous flux", and the mathematical structure of the equations.

MSC: 35M10; 35Q60; 76N10; 76N17; 76N20

Keywords: Compressible MHD; Vanishing shear viscosity limit; Boundary layer

1. Introduction

Magnetohydrodynamics (MHD) concerns the motion of a conducting fluid in an electromagnetic field with a very wide range of applications in astrophysics, plasma, and so on. The governing equations of MHD flows can be derived from fluid mechanics with appropriate modifications to account for electrical forces. Because the dynamic motion of the fluid and the

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magnetic field interact on each other and both the hydrodynamic and the electrodynamic effects are strongly coupled, the mathematical structural and dynamic mechanism of multi-dimensional MHD equations is considerably complicated. So, similarly to that in [1,2,26], we consider the following one-dimensional equations for compressible isentropic planar MHD flows:

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + P + \frac{1}{2}|\boldsymbol{b}|^2)_x = \lambda u_{xx}, \\ (\rho \boldsymbol{w})_t + (\rho u \boldsymbol{w} - \boldsymbol{b})_x = \mu \boldsymbol{w}_{xx}, \\ \boldsymbol{b}_t + (u\boldsymbol{b} - \boldsymbol{w})_x = \nu \boldsymbol{b}_{xx}, \end{cases} \tag{1.1}$$

where the unknown functions $\rho, u, \boldsymbol{w} = (w^1, w^2)$, $P, \boldsymbol{b} = (b^1, b^2)$ are the density of the fluid, the longitudinal velocity, the transverse velocity, the pressure, and the transverse magnetic field, respectively. The constants $\lambda > 0$, $\mu \geq 0$ are the bulk and the shear viscosity coefficients respectively, and the constant $\nu > 0$ is the resistivity coefficient acting as the magnetic diffusion coefficient of the magnetic field. We focus our interest on the case of isentropic MHD flows obeying the equation of state:

$$P(\rho) = A\rho^\gamma, \tag{1.2}$$

where $A > 0$ and $\gamma > 1$ are positive physical constants.

Without loss of generality, let $\Omega \triangleq (0, 1)$. In this paper, we shall study an initial-boundary value problem of (1.1), (1.2) with the initial and boundary data:

$$\begin{cases} (\rho, u, \boldsymbol{w}, \boldsymbol{b})|_{t=0} = (\rho_0, u_0, \boldsymbol{w}_0, \boldsymbol{b}_0)(x), \quad x \in (0, 1), \\ u(0, t) = u(1, t) = 0, \quad \boldsymbol{b}(0, t) = \boldsymbol{b}(1, t) = 0, \quad t \geq 0, \\ \boldsymbol{w}(0, t) = \boldsymbol{w}_1(t), \quad \boldsymbol{w}(1, t) = \boldsymbol{w}_2(t), \quad t \geq 0. \end{cases} \tag{1.3}$$

which satisfy the compatibility conditions:

$$u_0|_{x=0,1} = 0, \quad \boldsymbol{b}_0|_{x=0,1} = 0, \quad \boldsymbol{w}_0(0) = \boldsymbol{w}_1(0) \quad \text{and} \quad \boldsymbol{w}_0(1) = \boldsymbol{w}_2(0).$$

The main purpose of this paper is to investigate the asymptotic behavior of the solution as the shear viscosity goes to zero (i.e., $\mu \to 0$). Formally, if $\mu \equiv 0$, then (1.1) turns into:

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + P + \frac{1}{2}|\boldsymbol{b}|^2)_x = \lambda u_{xx}, \\ (\rho \boldsymbol{w})_t + (\rho u \boldsymbol{w} - \boldsymbol{b})_x = 0, \\ \boldsymbol{b}_t + (u\boldsymbol{b} - \boldsymbol{w})_x = \nu \boldsymbol{b}_{xx}, \end{cases} \tag{1.4}$$

with the following initial and boundary conditions:

$$\begin{cases} (\rho, u, \boldsymbol{w}, \boldsymbol{b})|_{t=0} = (\rho_0, u_0, \boldsymbol{w}_0, \boldsymbol{b}_0)(x), \quad x \in (0, 1), \\ u(0, t) = u(1, t) = 0, \quad \boldsymbol{b}(0, t) = \boldsymbol{b}(1, t) = 0, \quad t \geq 0, \end{cases} \tag{1.5}$$

where the pressure $P = P(\rho)$ fulfills the equation of state (1.2).