Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations 260 (2016) 3991–4015

Journal of Differential Equations

www.elsevier.com/locate/jde

Spreading in advective environment modeled by a reaction diffusion equation with free boundaries ☆

Hong Gu [a], Bendong Lou [b,*]

[a] *School of Applied Mathematics, Nanjing University of Finance & Economics, Nanjing 210023, China*
[b] *Department of Mathematics, Tongji University, Shanghai 200092, China*

Received 15 April 2015; revised 30 October 2015

Available online 14 November 2015

Abstract

We consider a reaction–diffusion–advection equation of the form: $u_t = u_{xx} - \beta u_x + f(u)$ for $x \in (g(t), h(t))$, where $g(t)$ and $h(t)$ are two free boundaries satisfying Stefan conditions, f is a bistable type of nonlinearity. This equation is used to describe the population dynamics in advective environment. We study the influence of the advection coefficient β on the dynamics of the solutions. We find a parameter $\beta^* > c_0$ (where $c_0 > 0$ is the speed of the traveling wave of the equation $u_t = u_{xx} + f(u)$) such that when $\beta \in (0, c_0)$ (resp. when $\beta \in (c_0, \beta^*)$), there is a vanishing-transition-spreading (resp. vanishing-transition-virtual spreading) trichotomy result for the long time behavior of the solutions; when $\beta \geqslant \beta^*$, all the solutions vanish (i.e., $h(t) - g(t)$ is bounded and $u \to 0$ uniformly).

MSC: 35K20; 35K55; 35R35; 35B40

Keywords: Reaction–diffusion–advection equation; Free boundary problem; Long time behavior; Spreading phenomenon

☆ This research was partly supported by NSFC (No. 11271285).
* Corresponding author.
E-mail addresses: honggu87@126.com (H. Gu), blou@tongji.edu.cn (B. Lou).

http://dx.doi.org/10.1016/j.jde.2015.11.002

1. Introduction

The study of spreading processes by using reaction diffusion equations traces back to the pioneering works of Fisher [16], and Kolmogorov, Petrovski and Piskunov [24]. In 1937, Fisher [16] introduced the equation $u_t = u_{xx} + u(1-u)$ for $x \in \mathbb{R}$ to model the spread of advantageous genetic trait in a population. On the basis of heuristic arguments, he found that there exist traveling wave solutions $u(t,x) = \phi(x - ct)$ connecting two steady states 1 and 0 for all speeds $c \geqslant 2$, and that there are no such traveling wave solutions for slower speed. In the same year, Kolmogorov, Petrovski and Piskunov [24] gave a mathematical treatment for this equation. They proved the convergence of the solutions starting from certain initial data to the traveling wave solution with speed $c = 2$. From then on, there have been extensive investigation on population dynamics by using reaction diffusion equations. For example, in the classical papers [2,3], Aronson and Weinberger gave a systematic investigation on the equation $u_t = u_{xx} + f(u)$ for $x \in \mathbb{R}$. When f is a monostable type of nonlinearity like $u(1-u)$, they proved the so-called *hair-trigger effect*, which says that *spreading* always happens (i.e. $\lim_{t\to\infty} u(t,x) = 1$) for the solution starting from any nonnegative and compactly supported initial data (no matter how small it is). When f is a bistable type of nonlinearity like $u(u - \frac{1}{3})(1-u)$, they gave some sufficient conditions for spreading and for *vanishing* (i.e. $\lim_{t\to\infty} u(t,x) = 0$). Furthermore, they showed that the way a spreading solution approaches 1 can be used to describe the spreading of a species, which is characterized by the traveling wave with minimal speed when f is monostable (or, the unique traveling wave when f is bistable), and the speed of this traveling wave determines the asymptotic spreading speed of the species.

In this paper we consider the population dynamics in advective environments, which means that the spreading of a species is affected by advection. In the field of ecology, organisms can often sense and respond to local environmental cues by moving towards favorable habitats, and these movement usually depend upon a combination of local biotic and abiotic factors such as stream, climate, food and predators. For example, some diseases spread along the wind direction. In 2009, Maidana and Yang [27] studied the propagation of West Nile Virus from New York City to California state. It was observed that West Nile Virus appeared for the first time in New York City in the summer of 1999. In the second year the wave front travels 187 km to the north and 1100 km to the south. Therefore, they took account of the advection movement and showed that bird advection becomes an important factor for lower mosquito biting rates. Another example is that Averill [4] considered the effect of intermediate advection on the dynamics of two-species competition system, and provided a concrete range of advection strength for the coexistence of two competing species. Moreover, three different kinds of transitions from small advection to large advection were illustrated theoretically and numerically. Many other examples involving advection were also found in the field of ecology (cf. [5,6,8,29,30,32–34] etc.).

From a mathematical point of view, to involve the influence of advection, one of the simplest but probably still realistic approaches is to assume that species can move up along the gradient of the density. The equation $u_t = u_{xx} - \beta u_x + f(u)$ with $\beta \in \mathbb{R}$ is such a simple example. Note that, in a moving coordinate frame $z = x - \beta t$, this equation reduces to the previous one: $w_t = w_{zz} + f(w)$ for $w(t,z) = u(t,x)$. Hence there is nothing new to be studied for the Cauchy problem. We will consider the equation in variable habitat with free boundaries.

In most spreading processes in the natural world, a spreading front can be observed. In one space dimension case, if the species initially occupies an interval $(-h_0, h_0)$, as time t increases from 0, it is natural to expect the end points of the habitat evolve into two spreading fronts: $x = g(t)$ on the left and $x = h(t)$ on the right. To determine how these fronts evolve with time,