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# Nonrelativistic Euler–Maxwell systems

Michael Sever

*Department of Mathematics, The Hebrew University, Givat Ram, Jerusalem 91904, Israel*

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**Abstract**

Results of two previous papers are used to reexamine Galilean symmetric Euler–Maxwell systems as candidate models of magnetohydrodynamic flow. For a single, electrically charged fluid, the results are largely negative. Under expected physical conditions, inclusion of the magnetic force on the fluid all but necessarily results in a modified Lundquist system. However the treatment is unsatisfactory in several respects.

The difficulty is circumvented, at the expense of additional complexity in the resulting system, in a plasma model with two fluid species of charge per unit mass of opposite sign. Under an additional physical assumption, the plasma model simplifies to the modified Lundquist system.


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## 1. Introduction

Simplification of a given mathematical model, approximation of a selected subset of solutions by the solutions of a simplified system, is familiar procedure. Based on previous results [8,9], we reconsider here the approximation of relativistic Euler–Maxwell systems by nonrelativistic systems, candidates for models of magnetohydrodynamic flow.

In this case the given, original system assumes the form of a dynamical system, a nonlinear, first-order system of balance laws in three space dimensions and time. Well-posedness of initial-boundary value problems for this system has not been established and is not immediately anticipated. Weak solutions, not uniquely determined by the initial-boundary data are anticipated,

*E-mail address:* sever@math.huji.ac.il.

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requiring a supplemental admissibility condition [6] to characterize physically acceptable solutions.

As is familiar, simplified systems are generally constructed by discard (including modification) of various terms from the original system. Here we place specific, requisite conditions on the systems to be so obtained, as a substitute for rigorous well-posedness results in justification of the approximating system. We tacitly assume the range of validity of any approximating system to be the region of phase space where the various discarded terms from the original system are negligibly small. Then a first requirement of any approximating system is existence of a usefully rich solution set within its range of validity. Here terms such as "negligibly small" and "usefully rich" will be left undefined as presumably specific application dependent.

Additionally, we expect approximating systems to reflect the entropy condition, symmetry group and whatever parameter limits of the original system.

The well-known nonrelativistic, compressible Euler system approximates the relativistic system in this sense, in the limits of fluid velocity and sound speed small compared with the speed of light. The thermodynamic entropy condition applies to both systems, so we shall require such for any system considered. The relativistic Euler system is Lorentz-rotation symmetric, whereas the compressible Euler system is Galilean-rotation symmetric, so Galilean-rotation symmetry is required of any nonrelativistic Euler–Maxwell systems so obtained.

Maxwell's equations are well-known as Lorentz symmetric. In a previous paper [8], two Galilean symmetric approximations of Maxwell's equations are constructed, one valid for small magnetic induction and one valid for small electric displacement. Each Maxwell approximation is equipped with supplemental equations describing balance of the momentum and energy of the electromagnetic field, but neither uniquely determines the electromagnetic field from given charge and current density. Thus the present discussion concerns coupling of one or more copies of the compressible Euler system with either of the two Maxwell approximations. Specifically, we discuss the unambiguous determination of the electromagnetic field and the choice of balance terms, maintaining the thermodynamic entropy, Galilean-rotation symmetry, and conservation of mass, momentum, energy and electric charge for the coupled Euler–Maxwell system so obtained. In addition, as a fluid becomes electrically neutral, interaction with the electromagnetic field disappears, and such should be reflected in an approximating system.

An important structural difference is anticipated between relativistic and nonrelativistic systems. Relativistic Euler–Maxwell systems are hyperbolic, and include lightspeed characteristics, which are necessarily absent in nonrelativistic systems. For this reason, we anticipate the loss of finite signal propagation speeds in the nonrelativistic systems constructed below.

As applied to a single, electrically charged fluid, the results of this analysis are largely negative. A model including the magnetic force on the fluid and with the desired structural features is constructed, but inclusion of the magnetic force is shown to be incompatible with the familiar expressions for the net charge and current density in terms of the fluid density and velocity and those for the force and work of the electromagnetic field on the fluid.

By inclusion of two fluid species with charge per unit mass of opposite sign, a model is constructed with only the physical assumptions of fluid velocities and sound speeds small in comparison with the speed of light and negligibly small electric displacement vector. Each fluid species is described by the familiar compressible Euler system with the expected balance terms, including the magnetic force in particular.

However, this system requires solution of a linear, inhomogeneous boundary-value problem at each point in time, to determine the electric field. Making an additional physical assumption, that the second fluid species is in quasi-equilibrium with the electromagnetic field, eliminates the