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J. Differential Equations 256 (2014) 2993–3010

Journal of Differential Equations

www.elsevier.com/locate/jde

Boundedness in quasilinear Keller–Segel systems of parabolic–parabolic type on non-convex bounded domains

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Received 14 November 2013; revised 7 January 2014

Available online 28 January 2014

Abstract

This paper deals with the quasilinear fully parabolic Keller–Segel system

$$\begin{cases} u_t = \nabla \cdot \big(D(u)\nabla u\big) - \nabla \cdot \big(S(u)\nabla v\big), & x \in \Omega,\ t > 0, \\ v_t = \Delta v - v + u, & x \in \Omega,\ t > 0, \end{cases}$$

under homogeneous Neumann boundary conditions in a bounded domain $\Omega \subset \mathbb{R}^N$ with smooth boundary, $N \in \mathbb{N}$. The diffusivity $D(u)$ is assumed to satisfy some further technical conditions such as algebraic growth and $D(0) \geqslant 0$, which says that the diffusion is allowed to be not only non-degenerate but also degenerate. The global-in-time existence and uniform-in-time boundedness of solutions are established under the subcritical condition that $S(u)/D(u) \leqslant K(u+\varepsilon)^\alpha$ for $u > 0$ with $\alpha < 2/N$, $K > 0$ and $\varepsilon \geqslant 0$. When $D(0) > 0$, this paper represents an improvement of Tao and Winkler [17], because the domain does not necessarily need to be convex in this paper. In the case $\Omega = \mathbb{R}^N$ and $D(0) \geqslant 0$, uniform-in-time boundedness is an open problem left in a previous paper [7]. This paper also gives an answer to it in bounded domains.
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MSC: primary 35K51; secondary 35B35

Keywords: Quasilinear degenerate Keller–Segel systems; Initial–boundary value problems; Boundedness

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1 Partially supported by Grant-in-Aid for Scientific Research (C), No. 25400119.

0022-0396/$ – see front matter © 2014 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.jde.2014.01.028

1. Introduction and results

The Keller–Segel system is proposed by Keller and Segel [11] in 1970. This system describes a part of the life cycle of cellular slime molds with chemotaxis. In more detail, slime molds move towards higher concentration of the chemical substance when they plunge into hunger. We denote by $u(x,t)$ the density of the cell population and by $v(x,t)$ the concentration of the signal substance at place x and time t. A number of variations of the original Keller–Segel system are proposed and studied (see Hillen and Painter [4]).

We consider the following quasilinear fully parabolic Keller–Segel system:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \nabla\cdot\big(D(u)\nabla u\big) - \nabla\cdot\big(S(u)\nabla v\big), & x\in\Omega,\ t>0,\\ \dfrac{\partial v}{\partial t} = \Delta v - v + u, & x\in\Omega,\ t>0,\\ \dfrac{\partial u}{\partial \nu} = 0, \quad \dfrac{\partial v}{\partial \nu} = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x), \quad v(x,0)=v_0(x), & x\in\Omega, \end{cases} \tag{KS}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary, $N\in\mathbb{N}$ and $\frac{\partial}{\partial\nu}$ denotes differentiation with respect to the outward normal of $\partial\Omega$. The initial data (u_0, v_0) is assumed to be a pair of non-negative functions. We suppose that $D, S \in C^2([0,\infty))$ and $D(0)\geqslant 0$: this means we allow the diffusion to be "degenerate". A quasilinear system such as (KS) was proposed by Painter and Hillen [13]. We note that the simplest choices $D(u)\equiv 1$ and $S(u)=u$ lead to the classical Keller–Segel system in [11].

From a mathematical point of view, it is meaningful question whether solutions remain *blow-up* or *bounded*. In the case of non-degenerate diffusion (with $D(0)>0$), Tao and Winkler [17] proved that solutions remain bounded under the condition that $S(u)/D(u)\leqslant K(u+1)^\alpha$ for $u>0$ with $\alpha<\frac{2}{N}$ and $K>0$, provided D satisfies the condition such as algebraic growth and Ω is *convex*. In particular, Tao [16] studied upon the choice $D(u)\equiv 1$ and proved the behavior in addition to boundedness of solutions on *convex* domains. It still remains to analyze on the following question:

$$\textit{Does boundedness of solutions to (KS) hold on "non-convex" domains?} \tag{Q1}$$

As to the special case $S(u)=u$, Senba and Suzuki [14] proved boundedness of solutions on non-convex domain. However, some additional assumptions were required on the initial data. In contrast, Winkler [18] showed that if $S(u)/D(u)\geqslant Ku^{\frac{2}{N}+\delta}$ for $u>1$ with $K>0$ and $\delta>0$, then there exist smooth solutions which blow up either in finite or infinite time, provided Ω is a ball. Indeed, Winkler [19] and Ciéslak and Stinner [1] asserted that the solutions blow up in finite time. In the case of degenerate diffusion (with $D(0)=0$), the model with $D(u)=u^{m-1}$ and $S(u)=u^{q-1}$ in $\mathbb{R}^N$ is studied by Sugiyama and Kunii [15] and in a previous work [7]. The existence of global-in-time weak solutions was shown when $q-m<0$ in [15] and when $q-m<\frac{2}{N}$ in [7]. Both results assert only for the global existence and leave the following question open:

$$\textit{Does boundedness of solutions to (KS) hold also for "degenerate" diffusion?} \tag{Q2}$$

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