

# Existence of capillary-gravity water waves with piecewise constant vorticity

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## Abstract

In this paper we construct small-amplitude periodic capillary-gravity water waves with a piecewise constant vorticity distribution. They describe water waves traveling on superposed linearly sheared currents that have different vorticities. This is achieved by associating to the height function formulation of the water wave problem a diffraction problem where we impose suitable transmission conditions on each line where the vorticity function has a jump. The solutions of the diffraction problem, found by using local bifurcation theory, are the desired solutions of the hydrodynamical problem.

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## 1. Introduction

We are concerned in this paper with the existence of steady periodic rotational waves interacting with currents that possess a discontinuous vorticity distribution, a situation accounting for sudden changes in a current and whose numerical simulations have only recently been undertaken, see [27,28]. More precisely, we establish the existence of capillary-gravity waves propagating at constant speed over a flat bed and interacting with several vertically superposed and linearly sheared currents of different (constant) vorticities. On physical grounds we can

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justify this situation by the fact that rotational waves generated by wind possess a thin layer of high vorticity that is adjacent to the wave surface [41,43], while in the near bed region there may exist currents resulting from sediment transport along the ocean bed [42].

A rotational fluid is not only interesting as an intricate mathematical problem but also serves a very concrete physical situation since it models wave–current interactions among other phenomena [5,25,45]. The waves we consider here are two-dimensional, have an a priori unknown free surface, and the vorticity function is piecewise constant. Though the vorticity distribution considered in the context of pure gravity waves in [9] corresponds to a merely bounded vorticity function, being more general than ours, we have in addition to gravity also the surface tension as a restoring force. This has the effect of adding a second order term in the top boundary condition of the height function formulation of the problem, situation that makes the analysis more intricate. We enhance that surface tension appears in the dynamics of water waves in many physical situations one of which is that of wind blowing over a still fluid surface and giving rise to two-dimensional small amplitude wave trains driven by capillarity [26] which grow larger and turn into capillary-gravity waves.

In the irrotational regime, the local bifurcation picture was described in [22] for waves traveling over a fluid layer of finite depth, respectively in [23,24,44] for waves of infinite depth. A particular feature in the irrotational case and for waves with constant vorticity [34] is that sometimes a mode interacts with another one of half its size giving rise to waves with two crests within a period, so-called Wilton ripples. Capillary-gravity water waves with a constant vorticity and stagnation points have been shown to exist in [33] by using techniques related to the ones employed in [10] for pure gravity waves. Allowing for a general Hölder continuous vorticity distribution, the existence theory is further developed in [47] for flows without stagnation points. The methods presented herein were recently further developed to establish the existence of capillary-gravity flows with arbitrary bounded [40] and even unbounded vorticity [35]. It is worth to mention that the many properties of capillary-gravity water waves, such as the regularity of the streamlines and of the wave profile [3,17,18,37,40], or the description of particle trajectories within the fluid [16], were only recently considered (see [4,7,11,36] for the case when surface tension is neglected).

In this paper we consider a different context than in [47], namely that of waves with a step function like vorticity. In order to prove our result we use the height function formulation of the water wave problem which is obtained via the Dubreil-Jacotin transformation (see [9] for details) and which has the advantage that the original free boundary problem is rendered into a quasilinear elliptic problem in a fixed domain. While in [9] the authors worked with a weak formulation of this problem, we are not able to do so here. This is due to the fact that the top boundary condition is nonlinear and contains second order derivatives of the unknown. We overcome this difficulty by associating to the height function formulation a diffraction (or transmission) problem where we impose suitable transmission conditions on each horizontal line where the vorticity has a jump. Then, using existing results for diffraction problems together with a Fourier multiplier argument, we are able to recast the mathematical problem as an abstract bifurcation problem in a functional analytic context which enables us to use local bifurcation theory. One of the difficulties in doing this is due to the lack of rigorous results concerning the  $C^{2+\alpha}$ -regularity of solutions to diffraction problems close to the interface where transmission conditions are imposed. We emphasize that diffraction problems are not seldom, they appear when multiphase flows are considered, as is the case of the Muskat problem [13]. The solutions that we find solve the boundary conditions of the problem in classical sense and the quasilinear equation in the weak sense defined in [9] and almost everywhere in the transformed fluid domain.

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