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Spectral analysis of the Schrödinger operator on binary tree-shaped networks and applications

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Abstract

In this paper we analyse the spectrum of the dissipative Schrödinger operator on binary tree-shaped networks. As applications, we study the stability of the Schrödinger system using a Riesz basis as well as the transfer function associated to the system.

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Contents

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1. Introduction

First of all, we introduce some notation needed to formulate the problem under consideration, which is simply that of [16]. We refer to [16] for more details.

To construct the binary tree $\mathcal{T}$ which will be considered in the following, we need some definitions (recall that a tree is a planar connected graph without paths).

A multi-index $\bar{\alpha}$ is a k-tuple $(\alpha_1, \ldots, \alpha_k)$ if k lies in $\mathbb{N} - \{0\}$ and it is empty if $k = 0$. For a fixed integer n, we choose for I the set of multi-indices $\bar{\alpha}$, with length k in $\{0, 1, \ldots, n\}$, such that, if $k \neq 0$, $\alpha_j \in \{1; 2\}$, for all j in $\{1, \ldots, k\}$. Then the set of vertices V of the tree $\mathcal{T}$ is $V := (\cup_{\bar{\alpha} \in I} \mathcal{O}_{\bar{\alpha}}) \cup \{\mathcal{R}\}$ where $\mathcal{R}$ is an additional vertex which will be the root of the tree $\mathcal{T}$.

The edges are denoted by $e_{\bar{\alpha}}$ with $\bar{\alpha}$ in I. Note that the number of edges is the cardinal of I and it holds: $|I| = N = 2^{n+1} - 1$.

Define, for any non-empty multi-indices $\bar{\alpha} = (\alpha_1, \ldots, \alpha_k)$ and $\bar{\beta} = (\beta_1, \ldots, \beta_m)$, the multi-index $\bar{\alpha} \circ \bar{\beta} := (\alpha_1, \ldots, \alpha_k, \beta_1, \ldots, \beta_m)$ of length $(k + m)$. Then, for a non-empty multi-index $\bar{\alpha} = (\alpha_1, \ldots, \alpha_k)$, the edge $e_{\bar{\alpha}}$ is chosen to have the extremities $\mathcal{O}_{\bar{\alpha}}$ and $\mathcal{O}_{\bar{\alpha'}}$ with $\bar{\alpha} = \bar{\alpha'} \circ (\alpha_k)$ and the edge e (corresponding to the case $\bar{\alpha} = \varnothing$) has the extremities $\mathcal{R}$ and $\mathcal{O}$.

See Fig. 1 for a representation in the case $n = 2$.

By the multiplicity of a vertex of $\mathcal{T}$ we mean the number of edges that branch out from that vertex. If the multiplicity is equal to one, the vertex is called exterior. Otherwise, it is said to be interior. We denote by *Int* the set of the interior vertices of the tree $\mathcal{T}$ and by *Dir* the set of the exterior vertices, except $\mathcal{R}$, which has a particular status in our problem. *Dir* is chosen for Dirichlet (see the problem below). A dissipation law is imposed at the root $\mathcal{R}$ which explains why it is isolated from the other exterior vertices.

Define

$$I_{Int} = \{\bar{\alpha};\ \mathcal{O}_{\bar{\alpha}} \in Int\},\ I_{Dir} = \{\bar{\alpha};\ \mathcal{O}_{\bar{\alpha}} \in Dir\}$$

which are the sets of the indices of the interior and exterior vertices, except $\mathcal{R}$, respectively.

Note that the multiplicity of each interior point of the tree $\mathcal{T}$ is equal to 3 and that the integer $(n + 1)$ represents the maximum level of the binary tree $\mathcal{T}$.

Furthermore, the length of the edge $e_{\bar{\alpha}}$ is equal to 1. Then, $e_{\bar{\alpha}}$ will be parametrized by its arc length by means of the functions $\pi_{\bar{\alpha}}$, defined in $[0, 1]$ such that $\pi_{\bar{\alpha}}(0) = \mathcal{O}_{\bar{\alpha}}$ and $\pi_{\bar{\alpha}}(1)$ is the other vertex of this edge. This choice seems unconventional but it is made for technical reasons.

In this paper, we study the dissipative Schrödinger operator under the tree-shaped network $\mathcal{T}$ introduced above. The case $N \geq 3$ is the one we are interested in: it corresponds to $n \geq 1$. The case $N = 1$ is well-known. See [19] and [16] concerning the model.