



Uniqueness of the self-similar profile for a kinetic annihilation model

Véronique Bagland ^a, Bertrand Lods ^{b,*}

^a Clermont Université, Université Blaise Pascal, Laboratoire de Mathématiques, CNRS UMR 6620, BP 10448, F-63000 Clermont-Ferrand, France

^b Università degli Studi di Torino & Collegio Carlo Alberto, Department of Economics and Statistics, Corso Unione Sovietica, 218/bis, 10134 Torino, Italy

Received 26 June 2014; revised 13 May 2015

Available online 2 September 2015

Abstract

We prove the uniqueness of the self-similar profile solution for a modified Boltzmann equation describing probabilistic ballistic annihilation. Such a model describes a system of hard spheres such that, whenever two particles meet, they either annihilate with probability $\alpha \in (0, 1)$ or they undergo an elastic collision with probability $1 - \alpha$. The existence of a self-similar profile for α smaller than an explicit threshold value $\underline{\alpha}_1$ has been obtained in our previous contribution [6]. We complement here our analysis of such a model by showing that, for some α^\sharp explicit, the self-similar profile is unique for $\alpha \in (0, \alpha^\sharp)$.

© 2015 Elsevier Inc. All rights reserved.

Keywords: Boltzmann equation; Ballistic annihilation; Self-similar profile; Uniqueness

Contents

1.	Introduction	7013
1.1.	Self-similar solutions	7014
1.2.	Strategy of proof and organization of the paper	7016
1.3.	Notations	7019
2.	A posteriori estimates on ψ_α	7019

* Corresponding author.

E-mail addresses: Veronique.Bagland@math.univ-bpclermont.fr (V. Bagland), bertrand.lods@unito.it (B. Lods).

2.1.	Uniform moments estimates	7020
2.2.	High-energy tails for the steady solution	7023
2.3.	Regularity of the steady state	7025
2.4.	High-energy tails for difference of steady solutions	7031
3.	Uniqueness and convergence results	7038
3.1.	Boltzmann limit	7038
3.2.	Uniqueness	7040
3.3.	Quantitative version of the uniqueness result	7049
	Acknowledgments	7050
	Appendix A. Regularity properties of \mathcal{Q}^+ revisited	7050
	A.1. Carleman representation	7051
	A.2. Regularity properties for cut-off collision kernels	7052
	A.3. Regularity properties for hard-spheres collision kernel	7056
	Appendix B. Useful interpolation inequalities	7057
	References	7058

1. Introduction

We investigate in the present paper a kinetic model, recently introduced in [9,12–15,21], which describes the so-called probabilistic ballistic annihilation of hard-spheres. In such a description, a system of (elastic) hard spheres interact according to the following mechanism: they freely move between collisions while, whenever two particles meet, they either annihilate with probability $\alpha \in (0, 1)$ or they undergo an elastic collision with probability $1 - \alpha$. In the spatially homogeneous situation, the velocity distribution $f(t, v)$ of particles with velocity $v \in \mathbb{R}^d$ ($d \geq 2$) at time $t \geq 0$ satisfies the following

$$\partial_t f(t, v) = (1 - \alpha)\mathcal{Q}(f, f)(t, v) - \alpha\mathcal{Q}^-(f, f)(t, v) \tag{1.1}$$

where \mathcal{Q} is the quadratic Boltzmann collision operator defined by

$$\mathcal{Q}(g, f)(v) = \int_{\mathbb{R}^d \times \mathbb{S}^{d-1}} |v - v_*| \left(g(v')f(v'_*) - g(v)f(v_*) \right) \frac{dv_* d\sigma}{|\mathbb{S}^{d-1}|},$$

where the post-collisional velocities v' and v'_* are parametrized by

$$v' = \frac{v + v_*}{2} + \frac{|v - v_*|}{2} \sigma, \quad v'_* = \frac{v + v_*}{2} - \frac{|v - v_*|}{2} \sigma, \quad \sigma \in \mathbb{S}^{d-1}.$$

The above collision operator $\mathcal{Q}(g, f)$ splits as $\mathcal{Q}(g, f) = \mathcal{Q}^+(g, f) - \mathcal{Q}^-(g, f)$ where the gain part \mathcal{Q}^+ is given by

$$\mathcal{Q}^+(g, f)(v) = C_d \int_{\mathbb{R}^d \times \mathbb{S}^{d-1}} |v - v_*| f(v'_*)g(v') dv_* d\sigma$$

Download English Version:

<https://daneshyari.com/en/article/4609793>

Download Persian Version:

<https://daneshyari.com/article/4609793>

[Daneshyari.com](https://daneshyari.com)