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Uniqueness of the self-similar profile for a kinetic annihilation model

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Abstract

We prove the uniqueness of the self-similar profile solution for a modified Boltzmann equation describing probabilistic ballistic annihilation. Such a model describes a system of hard spheres such that, whenever two particles meet, they either annihilate with probability $\alpha \in (0, 1)$ or they undergo an elastic collision with probability $1 - \alpha$. The existence of a self-similar profile for α smaller than an explicit threshold value $\underline{\alpha}_1$ has been obtained in our previous contribution [6]. We complement here our analysis of such a model by showing that, for some $\alpha^\sharp$ explicit, the self-similar profile is unique for $\alpha \in (0, \alpha^\sharp)$.

Keywords: Boltzmann equation; Ballistic annihilation; Self-similar profile; Uniqueness

Contents

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1. Introduction

We investigate in the present paper a kinetic model, recently introduced in [9,12–15,21], which describes the so-called probabilistic ballistic annihilation of hard-spheres. In such a description, a system of (elastic) hard spheres interact according to the following mechanism: they freely move between collisions while, whenever two particles meet, they either annihilate with probability $\alpha \in (0, 1)$ or they undergo an elastic collision with probability $1 - \alpha$. In the spatially homogeneous situation, the velocity distribution $f(t, v)$ of particles with velocity $v \in \mathbb{R}^d$ ($d \geqslant 2$) at time $t \geqslant 0$ satisfies the following

$$\partial_t f(t, v) = (1 - \alpha)\mathcal{Q}(f, f)(t, v) - \alpha \mathcal{Q}^-(f, f)(t, v) \tag{1.1}$$

where $\mathcal{Q}$ is the quadratic Boltzmann collision operator defined by

$$\mathcal{Q}(g, f)(v) = \int_{\mathbb{R}^d \times \mathbb{S}^{d-1}} |v - v_*| \Big(g(v') f(v'_*) - g(v) f(v_*) \Big) \frac{\mathrm{d}v_* \,\mathrm{d}\sigma}{|\mathbb{S}^{d-1}|},$$

where the post-collisional velocities v' and v'_* are parametrized by

$$v' = \frac{v + v_*}{2} + \frac{|v - v_*|}{2}\sigma, \qquad v'_* = \frac{v + v_*}{2} - \frac{|v - v_*|}{2}\sigma, \qquad \sigma \in \mathbb{S}^{d-1}.$$

The above collision operator $\mathcal{Q}(g, f)$ splits as $\mathcal{Q}(g, f) = \mathcal{Q}^+(g, f) - \mathcal{Q}^-(g, f)$ where the gain part $\mathcal{Q}^+$ is given by

$$\mathcal{Q}^+(g, f)(v) = C_d \int_{\mathbb{R}^d \times \mathbb{S}^{d-1}} |v - v_*| f(v'_*) g(v') \,\mathrm{d}v_* \,\mathrm{d}\sigma$$