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J. Differential Equations 259 (2015) 7260–7283

Journal of Differential Equations

www.elsevier.com/locate/jde

Solutions of fourth-order parabolic equation modeling thin film growth

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Received 29 December 2014; revised 24 July 2015

Available online 6 September 2015

Abstract

In this paper we study the well-posedness for a fourth-order parabolic equation modeling epitaxial thin film growth. Using Kato's Method [1–3] we establish existence, uniqueness and regularity of the solution to the model, in suitable spaces, namely $C^0([0,T]; L^p(\Omega))$ where $p = \frac{n\alpha}{2-\alpha}$ with $1 < \alpha < 2$, $n \in \mathbb{N}$ and $n \geq 2$. We also show the global existence solution to the nonlinear parabolic equations for small initial data. Our main tools are L_p–L_q-estimates, regularization property of the linear part of $e^{-t\Delta^2}$ and successive approximations. Furthermore, we illustrate the qualitative behavior of the approximate solution through some numerical simulations. The approximate solutions exhibit some favorable absorption properties of the model, which highlight the stabilizing effect of our specific formulation of the source term associated with the upward hopping of atoms. Consequently, the solutions describe well some experimentally observed phenomena, which characterize the growth of thin film such as grain coarsening, island formation and thickness growth.

MSC: 35A01; 35A02; 35A35; 35K55

Keywords: Thin-film equation; Scaling invariance; L_p–L_q estimates; Kato's method; Mild solution; Large time behavior

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http://dx.doi.org/10.1016/j.jde.2015.08.022
0022-0396/

1. Introduction

In the late 20th century, technologies based on the use of specific properties of thin film growth have known a significant development at the point they became one of the most important areas for improvement in the manufacturing of electronic components such as photovoltaic cells, due to their insulating or conductive properties. In the present work, we are concerned with the continuum model for epitaxial thin film growth which has been extensively used in the past [4,5]. Indeed, the compositions like $YBa_2Cu_3O_{7-\delta}$ (YBCO) are expected to be high-temperature super-conducting and could be used in the design of semi-conductors. Due to stringent tolerances on filter characteristics, the YBCO films must be highly uniform in thickness and texture [4]. The process of growing a thin film layer may be extremely complex and the development of experimental and mathematical tools for their study remains a focal point of research.

From the mathematical point of view, if u represents the scaled film height, the growth process may be formulated in the form [6,4,5]:

Find $u(x,t) \in \Omega \times (0,T)$ such that

$$\begin{cases} u_t + \Delta^2 u = \nabla \cdot f(\nabla u), \\ \partial_\nu u \mid_{\partial\Omega} = \partial_\nu \Delta u \mid_{\partial\Omega} = 0, \\ u(0) = \varphi, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ is a bounded smooth domain, $\partial\Omega$ denotes the boundary of Ω and ν is the unit outer vector normal to Ω. The term $\Delta^2 u$ denotes the capillarity-driven surface diffusion whereas $\nabla \cdot f(\nabla u)$ denotes the upward hopping of atoms.

Throughout this paper, we will assume that $f \in \mathbb{C}^1(\mathbb{R}^n, \mathbb{R}^n)$ with $f(0) = Df(0) = 0$ and for some $\alpha > 1$, and f satisfies the following growth condition: $\forall \xi_1, \xi_2 \in \mathbb{R}^n$

$$|f'(\xi_1) - f'(\xi_2)| \leq C(|\xi_1|^{\alpha-1} + |\xi_2|^{\alpha-1})|\xi_1 - \xi_2|. \tag{1.2}$$

As a simple example of Eq. (1.1), we take $f(\xi) = |\xi|^\alpha \xi$. To explain the meaning of the result, conceptually it is convenient to recall the dimensional analysis of Eq. (1.1). In fact the result is built upon a scaling invariance of the problem. Suppose $u(x,t)$ is a smooth solution to Eq. (1.1) in $\mathbb{R}^n$, then for each $\lambda > 0$,

$$u_\lambda(x,t) = \lambda^{\frac{2}{\alpha}-1} u(\lambda x, \lambda^4 t)$$

also solves (1.1) unless we consider the initial condition. We observe the following scaling identity

$$\|u_\lambda(\cdot,t)\|_{L^p(\mathbb{R}^n)} = \lambda^{\frac{2}{\alpha}-1-\frac{n}{p}} \left\| u(\cdot, \lambda^4 t) \right\|_{L^p(\mathbb{R}^n)}$$

or more generally, for $k \in \mathbb{N}$,

$$\left\| \nabla^k u_\lambda(\cdot,t) \right\|_{L^p(\mathbb{R}^n)} = \lambda^{\frac{2}{\alpha}+(k-1)-\frac{n}{p}} \left\| \nabla^k u(\cdot, \lambda^4 t) \right\|_{L^p(\mathbb{R}^n)}.$$

When p is chosen such that