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Weak stability for coupled wave and/or Petrovsky systems with complementary frictional damping and infinite memory

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Abstract

In this paper, we consider coupled wave–wave, Petrovsky–Petrovsky and wave–Petrovsky systems in N-dimensional open bounded domain with complementary frictional damping and infinite memory acting on the first equation. We prove that these systems are well-posed in the sense of semigroups theory and provide a weak stability estimate of solutions, where the decay rate is given in terms of the general growth of the convolution kernel at infinity and the arbitrary regularity of the initial data. We finish our paper by considering the uncoupled wave and Petrovsky equations with complementary frictional damping and infinite memory, and showing a strong stability estimate depending only on the general growth of the convolution kernel at infinity.

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1. Introduction

Let $g : \mathbb{R}_+ \to \mathbb{R}_+$ be a given function, $N \in \mathbb{N}^*$, $\Omega \subset \mathbb{R}^N$ be an open bounded domain with smooth boundary Γ, and $H = L^2(\Omega)$ be endowed with its natural inner product and corresponding norm denoted, respectively, by $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$. Let a, $\tilde{a}$, b, $\tilde{b}$ and d be variable coefficients depending only on the space variable such that $d, \tilde{b} \in L^\infty(\Omega)$,

$$(a, \tilde{a}, b) \in \begin{cases} W^{1,\infty}(\Omega) \times W^{1,\infty}(\Omega) \times W^{1,\infty}(\Omega): & \text{wave–wave,} \\ W^{2,\infty}(\Omega) \times W^{2,\infty}(\Omega) \times W^{2,\infty}(\Omega): & \text{Petrovsky–Petrovsky,} \\ W^{1,\infty}(\Omega) \times W^{2,\infty}(\Omega) \times W^{1,\infty}(\Omega): & \text{wave–Petrovsky,} \end{cases}$$

$$\inf_\Omega a > 0, \quad \inf_\Omega \tilde{a} > 0, \quad \inf_\Omega b \geq 0 \quad \text{and} \quad \inf_\Omega d \geq 0.$$

We consider the linear bounded self-adjoint operators $D = d\, Id$ and $\tilde{B} = \tilde{b}\, Id$ (Id is the identity operator), and the linear unbounded self-adjoint ones

$$(A, \tilde{A}, B) = \begin{cases} (-div(a\nabla), -div(\tilde{a}\nabla), -div(b\nabla)): & \text{wave–wave,} \\ (\Delta(a\Delta), \Delta(\tilde{a}\Delta), \Delta(b\Delta)): & \text{Petrovsky–Petrovsky,} \\ (-div(a\nabla), \Delta(\tilde{a}\Delta), -div(b\nabla)): & \text{wave–Petrovsky} \end{cases}$$

with domains $D(D) = D(\tilde{B}) = H$ and

$$(D(A), D(\tilde{A}), D(B)) \\ = \begin{cases} \left(H^2(\Omega) \cap H_0^1(\Omega), H^2(\Omega) \cap H_0^1(\Omega), H^2(\Omega) \cap H_0^1(\Omega)\right): & \text{wave–wave,} \\ \left(H^4(\Omega) \cap H_0^2(\Omega), H^4(\Omega) \cap H_0^2(\Omega), H^4(\Omega) \cap H_0^2(\Omega)\right): & \text{Petrovsky–Petrovsky,} \\ \left(H^2(\Omega) \cap H_0^1(\Omega), H^4(\Omega) \cap H_0^2(\Omega), H^2(\Omega) \cap H_0^1(\Omega)\right): & \text{wave–Petrovsky.} \end{cases}$$

Also

$$\left(D(A^{\frac{1}{2}}), D(\tilde{A}^{\frac{1}{2}}), D(B^{\frac{1}{2}}), D(\tilde{B}^{\frac{1}{2}}), D(D^{\frac{1}{2}})\right) \\ = \begin{cases} \left(H_0^1(\Omega), H_0^1(\Omega), H_0^1(\Omega), H, H\right): & \text{wave–wave,} \\ \left(H_0^2(\Omega), H_0^2(\Omega), H_0^2(\Omega), H, H\right): & \text{Petrovsky–Petrovsky,} \\ \left(H_0^1(\Omega), H_0^2(\Omega), H_0^1(\Omega), H, H\right): & \text{wave–Petrovsky.} \end{cases}$$

The aim of this paper is the study of the well-posedness and asymptotic behavior when time goes to infinity of solutions of the following coupled wave–wave, Petrovsky–Petrovsky and wave–Petrovsky system:

$$\begin{cases} u_{tt}(t) + Au(t) + Du_t(t) - \int_0^{+\infty} g(s)Bu(t-s)ds + \tilde{B}v(t) = 0, & \forall t > 0, \\ v_{tt}(t) + \tilde{A}v(t) + \tilde{B}u(t) = 0, & \forall t > 0 \end{cases} \tag{1.1}$$