



Finite and infinite speed of propagation for porous medium equations with nonlocal pressure

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Abstract

We study a porous medium equation with fractional potential pressure:

$$\partial_t u = \nabla \cdot (u^{m-1} \nabla p), \quad p = (-\Delta)^{-s} u,$$

for $m > 1$, $0 < s < 1$ and $u(x, t) \geq 0$. The problem is posed for $x \in \mathbb{R}^N$, $N \geq 1$, and $t > 0$. The initial data $u(x, 0)$ is assumed to be a bounded function with compact support or fast decay at infinity. We establish existence of a class of weak solutions for which we determine whether the property of compact support is conserved in time depending on the parameter m , starting from the result of finite propagation known for $m = 2$. We find that when $m \in [1, 2)$ the problem has infinite speed of propagation, while for $m \in [2, 3)$ it has finite speed of propagation. In other words $m = 2$ is critical exponent regarding propagation. The main results have been announced in the note [29].

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1. Introduction

In this paper we study the following nonlocal evolution equation

$$\begin{cases} u_t(x, t) = \nabla \cdot (u^{m-1} \nabla p), & p = (-\Delta)^{-s} u, & \text{for } x \in \mathbb{R}^N, t > 0, \\ u(x, 0) = u_0(x) & & \text{for } x \in \mathbb{R}^N, \end{cases} \tag{1.1}$$

for $m > 1$ and $u(x, t) \geq 0$. When $s = 0$ the model coincides with the classical *Porous Medium Equation* (PME) $u_t = \Delta u^m = \nabla(mu^{m-1} \nabla u)$, where the pressure p depends linearly on the density function u according to the Darcy Law. In this model the pressure p takes into consideration

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