Available online at www.sciencedirect.com
ScienceDirect
J. Differential Equations 260 (2016) 1154–1199
Journal of Differential Equations
www.elsevier.com/locate/jde

Finite and infinite speed of propagation for porous medium equations with nonlocal pressure

Diana Stan [a,b], Félix del Teso [a], Juan Luis Vázquez [a,*]

[a] *Departamento de Matemáticas, Universidad Autónoma de Madrid, Campus de Cantoblanco, 28049 Madrid, Spain*
[b] *Research Institute ICUB, University of Bucharest, 050107 Bucharest, Romania*

Received 9 July 2015; revised 1 September 2015

Available online 1 October 2015

Abstract

We study a porous medium equation with fractional potential pressure:

$$\partial_t u = \nabla \cdot (u^{m-1} \nabla p), \quad p = (-\Delta)^{-s} u,$$

for $m > 1$, $0 < s < 1$ and $u(x,t) \geq 0$. The problem is posed for $x \in \mathbb{R}^N$, $N \geq 1$, and $t > 0$. The initial data $u(x,0)$ is assumed to be a bounded function with compact support or fast decay at infinity. We establish existence of a class of weak solutions for which we determine whether the property of compact support is conserved in time depending on the parameter m, starting from the result of finite propagation known for $m = 2$. We find that when $m \in [1,2)$ the problem has infinite speed of propagation, while for $m \in [2,3)$ it has finite speed of propagation. In other words $m = 2$ is critical exponent regarding propagation. The main results have been announced in the note [29].

MSC: 26A33; 35K65; 76S05

Keywords: Nonlinear fractional diffusion; Fractional Laplacian; Riesz potential; Existence of solutions; Finite/infinite speed of propagation

* Corresponding author.
E-mail addresses: diana.stan@uam.es (D. Stan), felix.delteso@uam.es (F. del Teso), juanluis.vazquez@uam.es (J.L. Vázquez).

http://dx.doi.org/10.1016/j.jde.2015.09.023

Contents

1. Introduction

In this paper we study the following nonlocal evolution equation

$$\begin{cases} u_t(x,t) = \nabla \cdot (u^{m-1} \nabla p), \quad p = (-\Delta)^{-s} u, & \text{for } x \in \mathbb{R}^N,\ t > 0, \\ u(x,0) = u_0(x) & \text{for } x \in \mathbb{R}^N, \end{cases} \tag{1.1}$$

for $m > 1$ and $u(x,t) \geq 0$. When $s = 0$ the model coincides with the classical *Porous Medium Equation* (PME) $u_t = \Delta u^m = \nabla(mu^{m-1}\nabla u)$, where the pressure p depends linearly on the density function u according to the Darcy Law. In this model the pressure p takes into consideration