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Existence of solutions for perturbed fractional p-Laplacian equations

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Abstract

The purpose of this paper is to investigate the existence of weak solutions for a perturbed nonlinear elliptic equation driven by the fractional p-Laplacian operator as follows:

$$(-\Delta)_p^s u + V(x)|u|^{p-2}u = \lambda a(x)|u|^{r-2}u - b(x)|u|^{q-2}u \quad \text{in } \mathbb{R}^N,$$

where λ is a real parameter, $(-\Delta)_p^s$ is the fractional p-Laplacian operator with $0 < s < 1 < p < \infty$, $p < r < \min\{q, p_s^*\}$ and $V, a, b : \mathbb{R}^N \to (0, \infty)$ are three positive weights. Using variational methods, we obtain nonexistence and multiplicity results for the above-mentioned equations depending on λ and according to the integrability properties of the ratio a^{q-p}/b^{r-p}. Our results extend the previous work of Autuori and Pucci (2013) [5] to the fractional p-Laplacian setting. Furthermore, we weaken one of the conditions used in their paper. Hence the results of this paper are new even in the fractional Laplacian case.

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1. Introduction

In this paper we deal with the following one-parameter elliptic equations:

$$(-\Delta)_p^s u + V(x)|u|^{p-2}u = \lambda a(x)|u|^{r-2}u - b(x)|u|^{q-2}u \text{ in } \mathbb{R}^N, \tag{1.1}$$

where $N > ps$ with $s \in (0,1)$ and $(-\Delta)_p^s$ is the fractional p-Laplace operator which, up to normalization factors, may be defined as

$$(-\Delta)_p^s u(x) = 2 \lim_{\varepsilon \searrow 0} \int_{\mathbb{R}^N \setminus B_\varepsilon(x)} \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{N+ps}} dy$$

for $x \in \mathbb{R}^N$, where $B_\varepsilon(x) := \{y \in \mathbb{R}^N : |x-y| < \varepsilon\}$. As for some recent results on the fractional p-Laplacian, we refer to for example [18,19,22] and the references therein.

Notice that when $p = 2$, problem (1.1) reduces to the following fractional Laplacian equations:

$$(-\Delta)^s u + V(x)u = \lambda a(x)|u|^{r-2}u - b(x)|u|^{q-2}u \text{ in } \mathbb{R}^N, \tag{1.2}$$

which can been seen as the fractional form of the following classical stationary Schrödinger equations:

$$-\Delta u + V(x)u = \lambda a(x)|u|^{r-2}u - b(x)|u|^{q-2}u \text{ in } \mathbb{R}^N. \tag{1.3}$$

For standing wave solutions of fractional Schrödinger equations in $\mathbb{R}^N$, we refer to [11,14,16, 20,21,26] and the references therein. Especially, models governed by unbounded potentials involving fractional Schrödinger equations have been investigated in the last years, see for instance [13,32].

However, all these papers deal with problems which are not directly comparable to problem (1.1). In fact, the present paper is inspired by the following works: *Alama* and *Tarantello* in [2] studied the following Dirichlet problem with indefinite weights:

$$-\Delta u - \lambda u = \omega(x)u^{q-1} - h(x)u^{r-1} \quad \text{in } \Omega,$$

where $\lambda \in \mathbb{R}$, $\Omega \subset \mathbb{R}^N$ $(N \geq 3)$ is a bounded domain with smooth boundary, the coefficients ω, $h \in L^1(\Omega)$ are nonnegative and $2 < q < r$. They first showed that the existence, nonexistence and multiplicity results depend on λ and the integrability of the ratio w^{r-1}/h^{q-1}. In [28], *Pucci* and *Rădulescu* first considered the following related problem in the whole space:

$$-\text{div}(|\nabla u|^{p-2}\nabla u) + u^{p-1} = \lambda u^{q-1} - h(x)u^{r-1} \quad \text{in } \mathbb{R}^N, \tag{1.4}$$

where $h > 0$ satisfies

$$0 < \int_{\mathbb{R}^N} h(x)^{q/(q-r)} dx < \infty,$$