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J. Differential Equations 260 (2016) 1717–1746

Journal of Differential Equations

www.elsevier.com/locate/jde

Travelling waves in nonlinear magneto-inductive lattices

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Received 17 April 2015; revised 14 September 2015

Available online 21 October 2015

Abstract

We consider a lattice equation modelling one-dimensional metamaterials formed by a discrete array of nonlinear resonators. We focus on periodic travelling waves due to the presence of a periodic force. The existence and uniqueness results of periodic travelling waves of the system are presented. Our analytical results are found to be in good agreement with direct numerical computations.

Keywords: Travelling wave; Lattice wave; Forced magneto-inductive lattice

1. Introduction

In this work, we consider [26]

$$\frac{d^2}{dt^2}(u_n - \lambda u_{n-1} - \lambda u_{n+1}) + \gamma \frac{d}{dt} u_n + u_n + \frac{d^2}{dt^2}(u_n^2 - \lambda u_{n-1}^2 - \lambda u_{n+1}^2) + \gamma \frac{d}{dt} u_n^2 - h(\omega t + pn) = 0 \tag{1}$$

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http://dx.doi.org/10.1016/j.jde.2015.09.043

where $\gamma \geq 0$, $\lambda, \omega > 0$, $p \neq 0$ are parameters and $h \in C(\mathbb{R}, \mathbb{R})$ is such that

$$h(z) = \sum_{k\in\mathbb{Z}} h_k \,\mathrm{e}^{k\imath z}, \qquad \sum_{k\in\mathbb{Z}} |h_k| < \infty.$$

The equation models the dynamics of electromagnetic waves in the so-called magneto-inductive metamaterials.

Metamaterials are artificial materials that are engineered to have properties that may not be found in nature [5]. The (structural rather than chemical) engineering is achieved by composing periodic inhomogeneities to create desirable effective behaviour. The invention ignited a new paradigm in electromagnetism, including cloaking devices [20] (see also [17] for a recent comprehensive review of electromagnetic manipulation enabled by metamaterials). Magnetic metamaterials are non-magnetic materials exhibiting magnetic properties in the Terahertz and optical frequencies. They were predicted theoretically in [12,28] and demonstrated experimentally in, e.g., [7,11,14,27]. When the materials composed of magnetic metamaterials, e.g., split-ring resonators (SRRs), are magnetically weakly coupled through their mutual inductances, one obtains magneto-inductive metamaterials [9,21–23]. Magneto-inductive metamaterials consisting of periodic arrays of SRRs have been made in one-, two- and three dimensions [19]. Both the dimensions of SRRs and their inter-space distance are small relative to the free space wavelength and, thus, the dynamics of electromagnetic fields in such settings are governed by the quasi magnetostatic approximation [13].

Model equations for the propagation of nonlinear electromagnetic waves in metamaterials can be grouped into two classes. The first approach assumes that effective metamaterials are homogeneous media with specific physical properties resulting in partial differential equations, such as coupled short-pulse equation [24] and higher-order nonlinear Schrödinger equations [25]. In the second class, metamaterials are modelled by arrays of coupled oscillators, i.e. lattice equations, such as a nonlinear Klein–Gordon equation [18] and coupled Klein–Gordon equations [15,16]. The governing equation (1) falls into the second approach with γ representing the loss coefficient, λ is the coupling parameter, h is an external forcing that is periodic in time and varying in the spatial direction.

In the lattice equation (1) the nonlinearity appears in the coupling terms and in the damping. This is due to the assumption of the nonlinearity of the capacitance of the split-ring resonators that compose the magneto-inductive materials [26]. Note that the nonlinearity is different from our previous work [1,6], where it is in the onsite potential. The present work is to study the effect of such nonlinearity. Here, we investigate the existence of travelling periodic solutions of system (1) due to the periodic forcing, and the bifurcation of such solutions with small γ, λ and h. We also study the modulational stability of the periodic solutions by computing Floquet multipliers of the linearized systems.

Note that in our governing equation (1) the nonlinearity in the coupling terms between the sites is akin to that in the Fermi–Pasta–Ulam lattices [10]. However, there is a significant difference in fact that the coupling in our governing equation is also the derivative term. The bifurcation structures of periodic solutions in general FPU lattices forced by periodic drive were studied in [8]. It is imperative to study the effect of the present nonlinear couplings to periodic solutions caused by the same periodic drive. Using Lyapunov–Schmidt reduction, we derive the asymptotic expressions of the bifurcating solutions.

The present paper is organized as follows. In Section 2 we are looking for a periodic travelling wave when its amplitude is limited by a function of the magnitude of the forcing. In the section,