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Existence of limit cycles and homoclinic bifurcation in a plant–herbivore model with toxin-determined functional response

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Abstract

In this paper we study a two-dimensional toxin-determined functional response model (TDFRM). The toxin-determined functional response explicitly takes into consideration the reduction in the consumption of plants by herbivore due to chemical defense, which generates more complex dynamics of the plant–herbivore interactions. The purpose of the present paper is to analyze the existence of limit cycles and bifurcations of the model. By applying the theories of rotated vector fields and the extended planar termination principle, we establish the conditions for the existence of limit cycles and homoclinic loop. It is shown that a limit cycle is generated in a supercritical Hopf bifurcation and terminated in a homoclinic bifurcation, as the parameters vary. Analytic proofs are provided for all results, which generalize the results presented in [11].

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Keywords: Plant–herbivore model; Limit cycles; Homoclinic bifurcation; Rotated vector fields; Extended planar termination principle

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1. Introduction

Over the past two decades, plant–herbivore interactions have been the focus of intensive research in ecology, evolutionary biology and resource management (see, for instance, [17,18,20] etc.). However, most of the models do not explicitly incorporate the toxin-mediated effects of browsing on plant population dynamics. To explore the impact of plant toxicity on the dynamics of plant–herbivore interactions, models with a toxin-determined functional response (TDFR) have been formulated, termed toxin-determined functional response model (TDFRM) [8,12]. Unlike the previous models of plant–mammal interactions (e.g., [1,10]), the TDFRM explicitly incorporates the reduction in herbivore growth due to plant toxins by modifying the traditional Holling Type II functional response.

A TDFRM can exhibit a variety of complex dynamics. For example, numerical studies in [12] show that a 3-dimensional TDFRM exhibits Hopf and period-doubling bifurcations. It is mathematically challenging to analyze a higher dimensional TDFRM; and hence, many results for those models are obtained via numerical simulations and analytical results are very limited. Even for a 2-dimensional TDFRM, such as the one considered in [11], the bifurcation analysis is done only partially. In particular, only local results are obtained for the existence of limit cycles and homoclinic bifurcation. The objective of the present paper is to extend these results of local existence to global analytically.

These results for the 2-dimensional TDFRM can be very useful when studying higher dimensional TDFRM. For example, to study biological questions associated with invasive species (see [6,9]), we can consider a 3-dimensional TDFRM and study the stability of a boundary state at which the resident species and herbivore coexist (either as a stable equilibrium or as a stable limit cycle). To apply available dynamical system tools (e.g., [19]), it requires the knowledge of the global dynamics of the boundary state, which is an interior state of a 2-dimensional TDFRM.

The 2-dimensional TDFRM system considered here includes only one plant species and one herbivore population [8,11,12]. Let $N = N(t)$ denote the biomass of plant available for herbivore and $P = P(t)$ denote the biomass of herbivores at time t. The 2-dimensional TDFRM system is described by the following equations

$$\begin{aligned} \frac{dN}{dt} &= rN\left(1-\frac{N}{K}\right) - C(N)P, \\ \frac{dP}{dt} &= \big(\beta C(N) - d\big)P, \end{aligned} \tag{1.1}$$

with toxin-determined functional response $C(N)$ given by

$$C(N) = f(N)\left(1-\frac{f(N)}{4G}\right), \tag{1.2}$$

where

$$f(N) = \frac{eN}{1+heN} \tag{1.3}$$

is the Holling Type II functional response. The parameters r and K are the intrinsic growth rate and carrying capacity, respectively, of the plant species; e is the rate of encounter per unit