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Traveling wave solutions for integro-difference systems

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Abstract

This paper is concerned with the traveling wave solutions for integro-difference systems of higher order. By using Schauder fixed point theorem, the existence of traveling wave solutions is reduced to the existence of generalized upper and lower solutions. Then the asymptotic behavior of traveling wave solutions is studied by the idea of contracting rectangles. To illustrate our results, the traveling wave solutions of three systems are considered, which completes some known results.

MSC: 45C05; 45M05; 92D40

Keywords: Generalized upper and lower solutions; Contracting rectangle; Asymptotic behavior; Asymptotic spreading

1. Introduction

In this paper, we investigate the existence and asymptotic behavior of traveling wave solutions of the following integro-difference system

$$u_{n+1}^i(x) = \int_{\mathbb{R}} P_i\big[u_{n-\tau+1}^1(y), \cdots, u_n^1(y), u_{n-\tau+1}^2(y), \cdots, u_n^m(y)\big] k_i(x-y)dy, \tag{1.1}$$

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in which $m \in \mathbb{N}$ and $\tau \in \mathbb{N}$ are constants, $i \in I := \{1, 2, \cdots, m\}$, $n \in \mathbb{N} \cup \{0\}$, $x \in \mathbb{R}$, $u_n^i \in \mathbb{R}$, $P_i : \mathbb{R}^{m\times\tau} \to \mathbb{R}$, $k_i : \mathbb{R} \to \mathbb{R}^+$ is a probability function or kernel function. Moreover, P_i satisfies the following assumptions:

(P1) there exists $\mathbf{M} = (M_1, M_2, \cdots, M_m)$ such that $[\mathbf{0}, \mathbf{M}]$ with $\mathbf{0} = (0, 0, \cdots, 0)$ is an invariant region of the corresponding difference system of (1.1), i.e.,

$$0 \le P_i[h_{11}, h_{12}, \cdots, h_{m\tau}] \le M_i$$

with

$$0 \le h_{ij} \le M_i, \quad i \in I,\ j \in J := \{1, 2, \cdots, \tau\};$$

(P2) there exists $L > 0$ such that

$$\left|P_i[h_{11}, h_{12}, \cdots, h_{m\tau}] - P_i[f_{11}, f_{12}, \cdots, f_{m\tau}]\right| \le L \sum_{l\in I, j\in J} |h_{lj} - f_{lj}|$$

for any $h_{lj}, f_{lj} \in [0, M_l]$, $i, l \in I$, $j \in J$;

(P3) $P_i[0, 0, \cdots, 0] = 0$ and there exists $\mathbf{E} = (E_1, E_2, \cdots, E_m)$ such that

$$P_i\big[\overbrace{E_1, \cdots, E_1}^{\tau}, \overbrace{E_2, \cdots, E_2}^{\tau}, E_3, \cdots, E_m\big] = E_i, \quad i \in I;$$

(P4) $\mathbf{0} \ll \mathbf{E} \le \mathbf{M}$.

Moreover, for every $i \in I$, the probability function k_i satisfies the following conditions:

(k1) $k_i : \mathbb{R} \to \mathbb{R}^+$ is Lebesgue measurable and integrable;
(k2) $k_i : \mathbb{R} \to \mathbb{R}^+$ satisfies $k_i(x) = k_i(-x)$, $x \in \mathbb{R}$;
(k3) $\int_{\mathbb{R}} k_i(y)dy = 1$ and $\int_{\mathbb{R}} k_i(y)e^{\lambda y}dy < \infty$ for any $\lambda \ge 0$.

If $m = 1$ and $\tau = 1$, then (1.1) becomes

$$v_{n+1}(x) = \int_{\mathbb{R}} b\big(v_n(y)\big)k(x-y)dy, \tag{1.2}$$

in which $b : \mathbb{R}^+ \to \mathbb{R}^+$ is bounded and continuous and $b(0) = 0$, $k : \mathbb{R} \to \mathbb{R}^+$ satisfies (k1)–(k3). In the past three decades, the traveling wave solutions of (1.2) have been widely studied, we refer to Creegan and Lui [1], Hsu and Zhao [2], Kot [3], Kot et al. [4], Liang and Zhao [6], Lui [11–14], Neubert and Caswell [15], Wang et al. [19], Weinberger [20,21], Weinberger et al. [22,23] and Yi et al. [25]. In these papers, the (local) monotonicity of b plays a very important role.

If $\tau = 1$, then Liang and Zhao [6], Weinberger et al. [22] and Yi et al. [25] investigated the propagation modes of (1.1) by traveling wave solutions and asymptotic spreading. Similar to the study of scalar equations, the monotonicity of semiflows (see [18]) is the most essential assumption in [6,22]. Recently, Lin and Li [8] and Lin et al. [9] considered the existence of traveling wave solutions of a competitive system ($\tau = 1$, $m = 2$) by a cross iteration scheme.