Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations 258 (2015) 2941–2965

Journal of Differential Equations

www.elsevier.com/locate/jde

Domain walls dynamics in a nanowire subject to an electric current

Gilles Carbou[a,*], Rida Jizzini[b]

[a] *Laboratoire de Mathématiques et Applications de Pau, UMR CNRS 5131, Université de Pau et des Pays de l'Adour, Avenue de l'Université, 64000 Pau cedex, France*

[b] *Institut de Mathématiques de Bordeaux, Université Bordeaux 1, 351 cours de la Libération, 33405 Talence cedex, France*

Received 3 November 2014; revised 8 December 2014

Available online 22 January 2015

Abstract

In this work, we aim to study a one dimensional model of ferromagnetic wire submitted to an electric field modeled by a transport term involved in the Landau–Lifschitz equation. We will consider two types of wires: the case of a wire with elliptical section and the case of a wire with round section. For both cases we prove the stability of exact solutions describing one wall configurations.

Keywords: Landau–Lifschitz equation; Ferromagnetic materials; Stability

---

1. Introduction

Ferromagnetic materials exhibit a strong attraction to magnetic fields. They are able to retain their magnetic properties after vanishing of the external field. This particularity gives them important properties for applications in many industrial sectors as radar protection, storage of information, energy management and telecommunications equipment (see [3,8,9] and [16] for more information).

---

[*] Corresponding author.
E-mail address: gilles.carbou@univ-pau.fr (G. Carbou).

http://dx.doi.org/10.1016/j.jde.2014.12.035

One of the most promising applications of ferromagnetic nanowires is the digital data storage in “racetrack memories” (see [14]). The formation of magnetic domains, in which the magnetization is along the wire, either in one sense or in the other sense, allows the storage of digital informations. The domains are separated by domain walls, thin zones in which the magnetization presents large variations. The information is transported along the wire (for example to a reading head) by an electric current inducing walls motion. Compared to an applied magnetic field, this solution can be very useful. Indeed it is easier to generate a constant electric current in a wire, even if it is not straight. Moreover, a constant applied current induces a motion of the walls preserving their positions one with respect to each other, while an application of a constant magnetic field in a finite wire can induce the collapse of consecutive walls and consequently the annihilation of domains.

In this paper we address the description of the effects of an electric current in a ferromagnetic material for a one dimensional model of infinite wire. In particular we will consider one wall configurations in the case of wires with round cross section or with elliptical cross section. For both cases, we will prove the stability of such configurations.

Let us describe the one dimensional model we deal with.

A ferromagnetic material is characterized by a spontaneous magnetization represented by a magnetic moment. We consider an infinite homogeneous nanowire assimilated to the real line $\mathbb{R}e_1$, where (e_1, e_2, e_3) is the canonical basis of $\mathbb{R}^3$. We denote by m the magnetization:

$$\begin{aligned} m : \mathbb{R}_+ \times \mathbb{R} &\to \mathbb{R}^3 \\ (t, x) &\mapsto m(t, x). \end{aligned}$$

The magnetic moment m, the magnetic induction B and the magnetic field H are linked by the following constitutive relation:

$$B = H + \overline{m}$$

where B and H are defined on the whole space $\mathbb{R}^3$ and where $\overline{m}$ is the extension of m by zero outside the ferromagnetic domain.

Furthermore we assume that the studied material is saturated, so that the magnetic moment m takes its value in S^2 the unit sphere of $\mathbb{R}^3$. In the case of a ferromagnetic nanowire submitted to an electric current, Thiaville, Miltat, Nakatani and Susuki have proposed in [15] a process to integrate electric current effect on ferromagnetic materials in the Landau–Lifschitz equation, adding a transport term of the form $(v \cdot \nabla)m + m \times ((v \cdot \nabla)m)$ modeling the electric current, where $v(t, x)$ is a vector field directed along the direction of electrons motion, with an amplitude proportional to the current density.

Therefore, in the case of a one dimensional model of nanowire, the behavior of magnetic moment m is described by the following Landau–Lifschitz type equation:

$$\frac{\partial m}{\partial t} = -m \times H_e(m) - m \times \big(m \times H_e(m)\big) + v\frac{\partial m}{\partial x} + m \times v\frac{\partial m}{\partial x}, \tag{1}$$

where H_e, the effective field derived from micromagnetism energy (see [1]) is given by:

$$H_e(m) = \frac{\partial^2 m}{\partial x^2} + h_d(m).$$