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# Uniqueness and stability of traveling waves for cellular neural networks with multiple delays

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## Abstract

In this paper, we investigate the properties of traveling waves to a class of lattice differential equations for cellular neural networks with multiple delays. Following the previous study [38] on the existence of the traveling waves, here we focus on the uniqueness and the stability of these traveling waves. First of all, by establishing the *a priori* asymptotic behavior of traveling waves and applying Ikehara's theorem, we prove the uniqueness (up to translation) of traveling waves $\phi(n-ct)$ with $c \le c_*$ for the cellular neural networks with multiple delays, where $c_* < 0$ is the critical wave speed. Then, by the weighted energy method together with the squeezing technique, we further show the global stability of all non-critical traveling waves for this model, that is, for all monotone waves with the speed $c < c_*$, the original lattice solutions converge time-exponentially to the corresponding traveling waves, when the initial perturbations around the monotone traveling waves decay exponentially at far fields, but can be arbitrarily large in other locations.


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*Keywords:* Cellular neural networks; Uniqueness; Asymptotic behavior; Nonlinear stability; Weighted energy method; Squeezing technique

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## 1. Introduction

Cellular neural networks (CNN) were first proposed by Chua and Yang [8,9] as an achievable alternative to fully-connected neural networks in electric circuit systems. Since then, the study of cellular neural networks has been one of hot research topics due to their many significant applications to a broad scope of problems arising from, for example, image and video signal processing, robotic and biological visions, and higher brain functions [7–9,29]. The infinite system of the ordinary differential equations for the one-dimensional CNN with a neighborhood of radius $m$ but without inputs is of the form

$$x_n'(t) = -x_n(t) + z + \sum_{i=1}^{m} a_i f(x_{n-i}(t)) + \alpha f(x_n(t)) + \sum_{i=1}^{m} \beta_i f(x_{n+i}(t)), \tag{1.1}$$

for $n \in \mathbb{Z}$, $m \in \mathbb{N}$. Here, $x_n(t)$ denotes the state function of cell $C_n$ at time $t$. The quantity $z$ is called a threshold or bias term and is related to independent voltage sources in electric circuits. The nonnegative constant coefficients $a_i$, $\alpha$ and $\beta_i$ of the output function $f$ constitute the so-called space-invariant template that measure the synaptic weights of self-feedback and neighborhood interactions. When the cells are taken account of the instantaneous self-feedback and neighborhood interaction with distributed delays, because of the finite switching speed of signal transmission, the dynamic system can be presented by the following nonlocal lattice differential equation with multi-delays [32,38]

$$\begin{aligned} x_n'(t) = -x_n(t) &+ \sum_{i=1}^{m} a_i \int_0^{\tau} J_i(y) f(x_{n-i}(t-y))dy + \alpha \int_0^{\tau} J_{m+1}(y) f(x_n(t-y))dy \\ &+ \sum_{j=1}^{l} \beta_j \int_0^{\tau} J_{m+1+j}(y) f(x_{n+j}(t-y))dy \end{aligned} \tag{1.2}$$

for $n \in \mathbb{Z}$, $m, l \in \mathbb{N}$, where $J_i : [0, \tau] \to [0, \infty)$ is the density function for delay effect of the neighbors. Particularly, if the kernels are taken as some delta-functions $J_i = \delta(y - \tau_i)$, $i = 1, 2, \cdots, m + l + 1$, where $\tau_i > 0$ are the time-delays, then the equation (1.2) is reduced to the following multiple time-delayed lattice differential equation for the cellular neural networks [12–16,19,30]

$$\begin{aligned} x_n'(t) = -x_n(t) &+ \sum_{i=1}^{m} a_i f(x_{n-i}(t-\tau_i)) + \alpha f(x_n(t-\tau_{m+1})) \\ &+ \sum_{j=1}^{l} \beta_j f(x_{n+j}(t-\tau_{m+1+j})), \end{aligned} \tag{1.3}$$

subjected to the initial data

$$x_n(s) = x_n^0(s), \quad s \in [-r, 0], \quad r = \max_{1 \le i \le m+1+l} \{\tau_i\}. \tag{1.4}$$