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Existence, stability and optimality for optimal control problems governed by maximal monotone operators ☆

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Abstract

We study optimal control problems governed by maximal monotone differential inclusions with mixed control-state constraints in infinite dimensional spaces. We obtain some existence results for this kind of dynamics and construct the discrete approximations that allows us to strongly approximate optimal solutions of the continuous-type optimal control problems by their discrete counterparts. Our approach allows us to apply our results for a wide class of mappings that are applicable in mechanics and material sciences.

MSC: 49J52; 49J53; 49K24; 49M25; 90C30

Keywords: Optimal control; Maximal monotone differential inclusions; Normal cone mappings; Discrete approximations; Variational analysis; Generalized differentiation

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1. Introduction

Nonautonomous differential inclusions of the form

$$-\dot{x}(t) \in \mathcal{A}_t(x(t)) \quad \text{for} \quad t \in \mathcal{I}, \quad \text{and} \quad x(t_0) = \mathrm{x}_0, \tag{1.1}$$

governed by maximal monotone operators, have an important number and variety of applications in partial differential equations (heat equations and obstacle problems), mechanics (rigid-body systems with impact, Coulomb friction), electricity (diodes and transistors) and management (queueing and use of limited resources), as extensively described in [4,6,22,29] and the references therein. Of particular interest is the case where, at every instant, the operator $\mathcal{A}_t$ is the subdifferential of a proper, lower-semicontinuous and convex function Φ_t,

$$-\dot{x}(t) \in \partial\Phi_t(x(t)) \quad \text{for} \quad t \in \mathcal{I}, \quad \text{and} \quad x(t_0) = \mathrm{x}_0,$$

in view of its applications in nonsmooth optimization and optimal control problems. It models, for instance, penalization or regularization procedures that yield constrained optima as time goes to $+\infty$ (see [2,7,12], among others). A special – yet very important – class of differential inclusions of this kind is the *sweeping process* introduced by J.-J. Moreau in the 1970s, given by

$$-\dot{x}(t) \in N(x(t); C(t)) \quad \text{for} \quad t \in \mathcal{I}, \quad \text{and} \quad x(t_0) = \mathrm{x}_0.$$

Here, $N(\cdot, C)$ is the *normal cone operator* with respect to a set C, and coincides with the subdifferential of the indicator function of C in the sense of Convex Analysis. Roughly speaking, it models the movement of a particle that is constrained to lie in a moving set, being forced to head inwards upon contact with the boundary. The sweeping process has many applications in evolutionary variational inequalities, see [6,15,22]. Recently, the sweeping process has gathered much attention in mathematical viewpoint, see [5,8,10,21,26,30].

Usually, the study of the aforementioned systems relies on context-dependent techniques. In the most general setting, namely (1.1), existence and approximation results often require some geometric regularity conditions that are not applicable to the sweeping process and other relevant particular instances [1,11,16,21,30]. This paper is a contribution towards a more unified approach, especially when (1.1) describes a *controlled* dynamical system. In other words, when the time-dependence is induced by an external action that affects the system.

1.1. Overview of our main results

Throughout this paper, H and K are real Hilbert spaces, $(t, \mathrm{x}, \mathrm{u}) \mapsto F(t, \mathrm{x}, \mathrm{u})$ is a set-valued mapping such that, for every $t \in \mathcal{I} := [0, T]$ and $\mathrm{u} \in \mathrm{K}$, $F(t, \cdot, \mathrm{u})$ is maximal monotone on H. We consider a controlled differential inclusion of the from

$$-\dot{x}(t) \in F(t, x(t), u(t)) \quad \text{a.e. } t \in [0, T], \quad \text{and} \quad x(0) = \mathrm{x}_0. \tag{1.2}$$

Here, $x : [0, T] \to \mathrm{H}$ and $u : [0, T] \to \mathrm{K}$ represent the trajectory and control of the dynamical system, respectively, and $\mathrm{x}_0 \in \mathrm{H}$ is the initial condition. The main relevant properties (for our purpose) of maximal monotone operators, along with some particular instances of (1.2), will be recalled in Section 2.