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Weak almost periodic motions, minimality and stability in impulsive semidynamical systems

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Abstract

In this paper, we study topological properties of semidynamical systems whose continuous dynamics are interrupted by abrupt changes of state. First, we establish results which relate various concepts as stability of Lyapunov, weakly almost periodic motions, recurrence and minimality. In the sequel, we study the stability of Zhukovskij for impulsive systems and we obtain some results about uniform attractors.
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1. Introduction

The theory of impulsive systems has been intensively investigated since this theory presents important applications. The reader may consult [1,9,13,14,16,23,24], for instance.

The present paper concerns with the topological study of semidynamical systems with impulses. We deal with results which encompass periodicity, recurrence, minimality, attractors and stability.

In [2], many recursive concepts (minimality, recurrence and almost periodic motions) are presented for continuous dynamical systems. Most of these results were generalized for impulsive systems, see [8,18,22] for example. However, many other properties of minimality, recurrence, weakly almost periodic motions, stability and attractors still need to be studied for the impulsive

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case. In this way, we present in this paper a study of recursive motions via stability theory (Lyapunov stability and Zhukovskij quasi stability). In the next lines we describe the organization of the paper and the main results.

We start by presenting a summary of the basis of the theory of semidynamical systems with impulse effects. In Section 2, we give some basic definitions and notations about impulsive semidynamical systems. In Section 3, we present some additional definitions and various results that will be very useful in the proof of the new results. Section 4 and Section 5 concern the main results of this paper. In the sequel, we mention some of these results.

In Section 4, we start by presenting the concept of weak almost $\tilde{\pi}$-periodic motions which was introduced by Saroop Kaul in [18]. We give sufficient conditions for a weakly almost $\tilde{\pi}$-periodic point to be $\tilde{\pi}$-recurrent, see Theorem 4.1. In Theorem 4.2, we show that a given point belongs to its limit set provided this point is weakly almost $\tilde{\pi}$-periodic and it does not belong to the impulsive set. The converse of Theorem 4.2 does not hold in general and we show a counterexample in Example 4.1. Following the results proved in the continuous case, we show that a $\tilde{\pi}$-recurrent point is weakly almost $\tilde{\pi}$-periodic provided it is Lyapunov $\tilde{\pi}$-stable, see Theorem 4.3.

In Theorem 4.4, we present sufficient conditions for the limit set of a given point to be minimal. Moreover, it is shown that if the points from the limit set which are outside of the impulsive set are Lyapunov $\tilde{\pi}$-stable, then these points are weakly almost $\tilde{\pi}$-periodic. Theorem 4.5 shows that the limit set of a point is minimal if and only if the trajectory of this point approaches uniformly to its limit set.

Section 5 deals with the quasi stability of Zhukovskij. In impulsive semidynamical systems, this kind of stability was introduced by Changming Ding in [15]. In Theorem 5.1, we show that the limit set of a given point is minimal provided the points of this limit set which are not in the impulsive set are Zhukovskij quasi $\tilde{\pi}$-stable. As consequence of this result, we give sufficient conditions for the points of a limit set which are not in the impulsive set to be $\tilde{\pi}$-recurrent, see Corollary 5.1.

Theorem 5.2 presents some conditions for a point to be periodic. In the last result, namely Theorem 5.3, we establish sufficient conditions for a limit set to be a uniform $\tilde{\pi}$-attractor.

2. Preliminaries

Let X be a metric space and $\mathbb{R}_+$ be the set of non-negative real numbers. The triple $(X, \pi, \mathbb{R}_+)$ is called a *semidynamical system*, if the function $\pi : X \times \mathbb{R}_+ \to X$ is continuous with $\pi(x, 0) = x$ and $\pi(\pi(x, t), s) = \pi(x, t + s)$, for all $x \in X$ and $t, s \in \mathbb{R}_+$. We denote such system simply by (X, π). For every $x \in X$, we consider the continuous function $\pi_x : \mathbb{R}_+ \to X$ given by $\pi_x(t) = \pi(x, t)$ and we call it the *motion* of x.

Let (X, π) be a semidynamical system. Given $x \in X$, the *positive orbit* of x is given by $\pi^+(x) = \{\pi(x, t): t \in \mathbb{R}_+\}$. Given $A \subset X$ and $t \geqslant 0$, we define

$$\pi^+(A) = \bigcup_{x \in A} \pi^+(x) \quad \text{and} \quad \pi(A, t) = \bigcup_{x \in A} \pi(x, t).$$

For $t \geqslant 0$ and $x \in X$, we define $F(x, t) = \{y \in X: \pi(y, t) = x\}$ and, for $\Delta \subset [0, +\infty)$ and $D \subset X$, we define

$$F(D, \Delta) = \bigcup \big\{F(x, t): x \in D \text{ and } t \in \Delta\big\}.$$

Then a point $x \in X$ is called an *initial point* if $F(x, t) = \emptyset$ for all $t > 0$.

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