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The Cauchy problem for the Ostrovsky equation with negative dispersion at the critical regularity

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Abstract

In this paper, we investigate the Cauchy problem for the Ostrovsky equation

$$\partial_x\left(u_t - \beta\partial_x^3 u + \frac{1}{2}\partial_x(u^2)\right) - \gamma u = 0,$$

in the Sobolev space $H^{-3/4}(\mathbf{R})$. Here $\beta > 0(< 0)$ corresponds to the positive (negative) dispersion of the media, respectively. P. Isaza and J. Mejía (2006) [13], (2009) [15], K. Tsugawa (2009) [26] proved that the problem is locally well-posed in $H^s(\mathbf{R})$ when $s > -3/4$ and ill-posed when $s < -3/4$. By using some modified Bourgain spaces, we prove that the problem is locally well-posed in $H^{-3/4}(\mathbf{R})$ with $\beta < 0$ and $\gamma > 0$. The new ingredient that we introduce in this paper is Lemmas 2.1–2.6.

MSC: 35G25

Keywords: Ostrovsky equation; Cauchy problem; Critical regularity; Dyadic bilinear estimates

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1. Introduction

In this paper, we consider the Ostrovsky equation

$$\partial_x\left(u_t-\beta\partial_x^3u+\frac{1}{2}\partial_x(u^2)\right)-\gamma u=0.$$

This equation is a mathematical model of the propagation of weakly nonlinear long waves in a rotating liquid. It was introduced by Ostrovsky in [27] as a model for weakly nonlinear long waves, by taking into account of the Coriolis force, to describe the propagation of surface waves in the ocean in a rotating frame of reference. The parameter γ is a positive number and measures the effect of rotation, and the parameter β is a nonzero real number of both signs and reflects the type of dispersion of the media. When $\beta<0$, the equation has negative dispersion and describes surface and internal waves in the ocean and surface waves in a shallow channel with an uneven bottom. When $\beta>0$, the equation has positive dispersion and describes capillary waves on the surface of liquid or for oblique magneto-acoustic waves (see [2,5,6]). In the absence of rotation (that is, $\gamma=0$), it becomes the Korteweg–de Vries equation. By changing variables the above Ostrovsky equation can be written in the form

$$u_t-\beta\partial_x^3u+\frac{1}{2}\partial_x(u^2)-\gamma\partial_x^{-1}u=0. \tag{1.1}$$

The Ostrovsky equation has many important properties, such as solitary waves or soliton solutions, etc., and it has closed relation to the KdV equation (see [22,23,26,30]). It draws much attention of physicists and mathematician. Many people have investigated the Cauchy problem for (1.1), for instance, see [7,9–17,21–23,28–30]. By using the Fourier restriction norm method introduced in [3,4], Isaza and Mejía [13] proved that (1.1) is locally well-posed in $H^s(\mathbf{R})$ with $s>-\frac{3}{4}$ in the negative dispersion case and is locally well-posed in $H^s(\mathbf{R})$ with $s>-\frac{1}{2}$ in the positive dispersion case. Later they showed the ill-posedness in $H^s(\mathbf{R})$ for $s<-\frac{3}{4}$ [15]. Recently, Tsugawa [26] proved the time local well-posedness in some anisotropic Sobolev space $H^{s,a}(\mathbf{R})$ with $s>-a/2-3/4$ and $0\le a\le 1$. The result includes the time local well-posedness in $H^s(\mathbf{R})$ with $s>-3/4$ for both positive and negative dispersion Ostrovsky equation. Thus, $s=-\frac{3}{4}$ is the critical regularity index for (1.1) in the both dispersion cases. Tsugawa considered also the weak rotation limit and proved that the solution of the Ostrovsky equation converges to the solution of the KdV equation when the rotation parameter γ goes to 0. However, the well-posedness of the Ostrovsky equation in the critical case has been still open.

In this paper we study the Cauchy problem for the Ostrovsky equation (1.1) with negative dispersion complimented with the initial condition

$$u(0,x)=u_0(x),\qquad x\in\mathbf{R}. \tag{1.2}$$

Compared with the KdV equation, the structure of the Ostrovsky equation is more complicated. More precisely, the phase function of the KdV equation is the smooth function ξ^3, while the phase function of the Ostrovsky equation is $\beta\xi^3+\frac{\gamma}{\xi}$, which has a singular point $\xi=0$. For the KdV equation, just as done in [1], two simple identities $a^3+b^3-\frac{(a+b)^3}{4}=\frac{3}{4}(a+b)(a-b)^2$ and $(a+b)^3-a^3-b^3=3ab(a+b)$ are valid to establish some key bilinear estimates, which guarantee the wellposedness in the critical space $H^s(\mathbf{R})$ with $s=-3/4$ (see Guo [8] and Kishimoto [20]).